

Variables & Patterns

Problem 1.2 - Graphing Data

Definitions:

variable - something that changes and can be measured

independent variable - (cause) determines the value of the other variable

dependent variable - (effect) value depends on the other variable

scale - interval used to label each axis

Steps For Graphing:

- 1.) select two variables
- 2.) determine which is the independent/dependent variable
label the x-axis with the independent variable
and the y-axis with the dependent variable
- 3.) select a scale for each axis
- 4.) plot the coordinates

EXAMPLE for jumping jack data-

independent variable - time

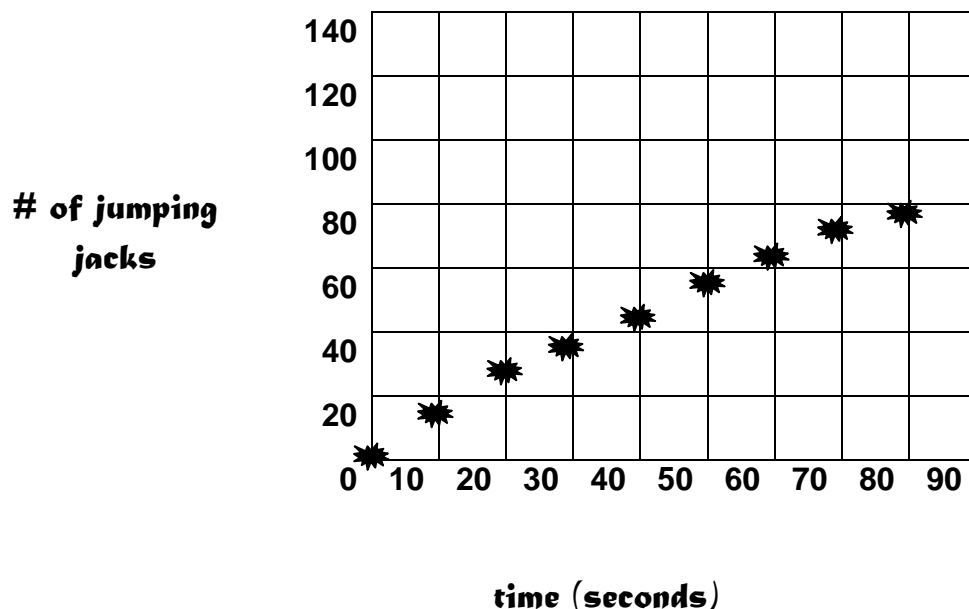
dependent variable - # of jumping jacks

x-scale 10

y-scale 20

time (seconds)	0	10	20	30	40	50	60	70	80
# jumping jacks	0	15	28	38	46	54	61	70	76

coordinates are (0,0) (10,15) (20,28) (30,38) and so on...



“The Bike Trip”

Variables & Patterns - Inv. 2

Ocean and History Bike Tours

created by
Sidney, Liz, Celia, Malcolm, and Theo

Day 1: Philadelphia to Atlantic City

The students begin their trip. Theo and Tony ride in the van to record the total distance traveled after each half-hour.

Day 2: Atlantic City to Lewes

The students travel for 5 hours to Cape May, then take a ferry to Lewes. Sidney and Sarah ride in the van to record distances.

Day 3: Lewes to Chincoteague Island

Celia collected data and recorded how far the riders were from Lewes each half-hour.

Day 4: Chincoteague Island to Norfolk

Malcolm and Sarah rode in the van, and made notes about the trip. No distances were recorded.

Day 5: Norfolk to Williamsburg

After lunch, the riders have a race.
Theo's bike records his speed every
10 minutes during the 90 minute race.

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Problem 2.1 - Bike Trip: Day 1

Creating a Written Report

1. Look at the data (see “The Bike Trip” handout, table of distances on day 1).
2. Look for patterns in the data.
3. Describe how the variables changed.
4. Give explanations for why those changes might have occurred.
5. Organize the information into sentences in a paragraph.

Example:

On day 1 of the trip we began in Philadelphia at 8:00 a.m. and traveled 89 miles to Atlantic City. The trip took 8 hours to complete. From 8:30 - 9:00 and from 10:00 - 10:30, we made the most progress. It was early and the weather was nice, so we were able to travel 10 miles in 30 minutes. In the afternoon we stopped for lunch, making the least progress from 11:00 - 12:00. Overall, we were able to bike further in the morning, going 47 miles in 4 hours. In the afternoon we were tired and were only able to travel 42 miles in the last 4 hours.

Advantages of Tables, Graphs, and Written Reports

Tables	Graphs	Reports
<i>data is more exact</i>	<i>gives a visual picture</i>	<i>includes details</i>

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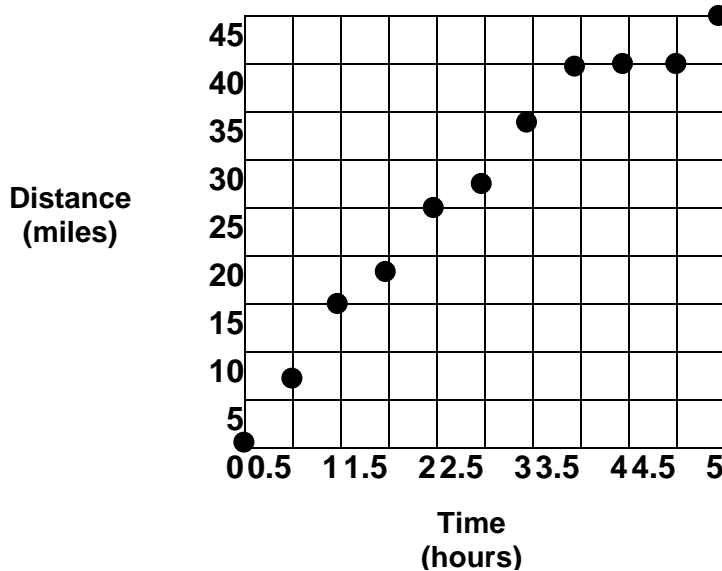
Problem 2.2 - Bike Trip: Day 2

Definitions:

coordinate pair - set of x and y values used to locate points on a graph

continuous - when the independent variable can be broken down into smaller, measurable units

A. Make a coordinate graph of the (time, distance) data given in the Day 2 table.



- B. How far did the group travel in one day? *45 miles*
How much time did it take them? *5 hours*
During which intervals did they make the most progress? *0.5 - 1.0 and 2.5 - 3.0*
During which intervals did they make the least progress? *3.5 - 4.5*
Did they go further in the morning or afternoon? *traveled furthest in the first 2.5 hrs*

- C. How do you find the intervals in which they made the least or most progress...
i) on a graph? *look at how far apart the coordinates are vertically - the bigger gap is when the most progress is made*
ii) in a table? *subtract each entry from the entry below it - the larger the difference, the more progress they made in that interval*

D. Which do you prefer when reading data, a table or a graph? Why?

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Problem 2.3 - Bike Trip: Day 3

A. Would it make sense to connect the points on this graph?

It would probably make sense to connect the points since time (our independent variable) is continuous.

B. Using the graph from “The Bike Trip” handout, Day 2, make a table of miles traveled after each half-hour.

Time (hours)	0	.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5
distance (miles)	0	7	13	22	22	30	22	31	36	48	48	56	63	72	74	81

C. What do you think happened between hours 2 and 4?

Between hours 1.5 and 2?

D. Which helps you see changes better, a table or a graph? Why?

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Problem 2.4 - Bike Trip: Day 4

Read through Malcolm and Sarah's notes from "The Bike Trip" handout - day 4.

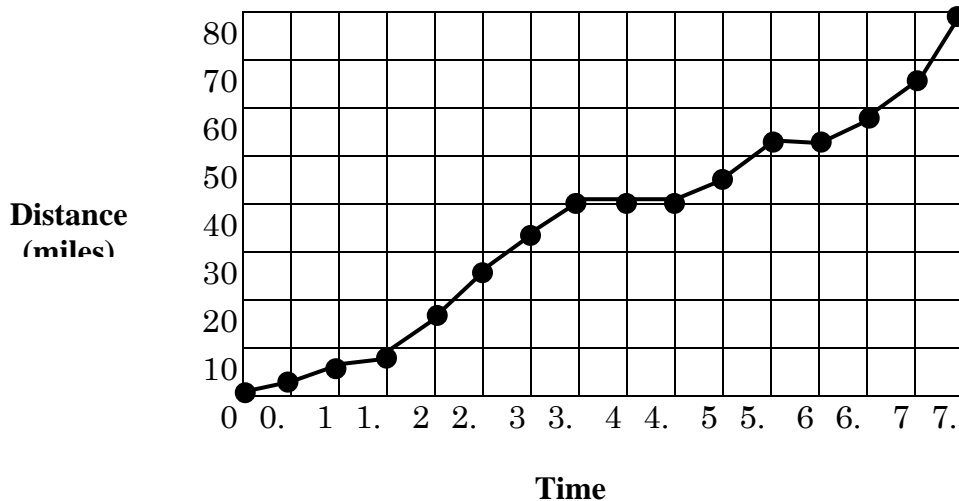
A. Make a table of data (time, distance) that reasonably fits the information in Malcolm and Sarah's notes. Start with the information you know and use common sense to fill in the remaining distances.

Let 8:30 be time 0, and increase by half hours (0, 0.5, 1, 1.5,...)

time (hours)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5
distance (miles)	0	3	5	8	18	25	33	40	40	40	46	52	52	59	65	80

B. Sketch a coordinate graph that shows the same information.

Chincoteague Island to Norfolk



Answer to Problem 2.4 Follow-Up

Use the notes from Malcolm and Sarah to make a table. Next, use half-hour intervals, because all the tables and graphs in this Investigation used that size time interval. Use the information that the trip started at 8:30 and that they traveled for 7.5 hours to fill in the time column of the table. Fill in the distance value to match the last time entry because the last note says they traveled 80 miles. The third note stated that the stop for lunch was about an hour and that they had traveled about halfway. Noon was a reasonable time for lunch (3.5 hours into the trip) and 40 would be the travel distance for that time because it is halfway. Use notes 1 & 2 to spread out the first 40 miles of the trip and notes 4 & 5 to spread out the last 40 miles of the trip. To make the graph, put time on the x-axis and distance on the y-axis and plot the data points from the table.

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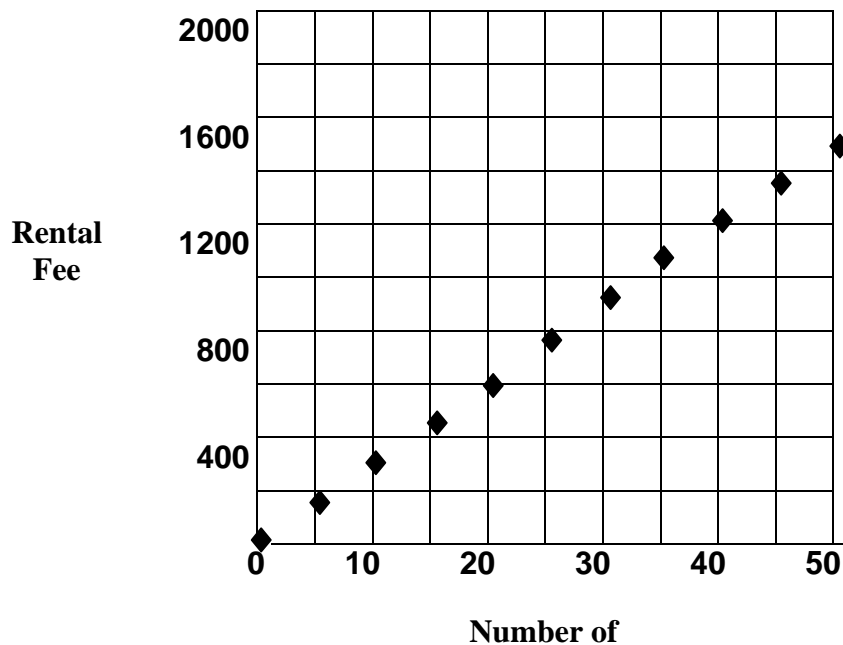
Problem 3.1 - Renting Bicycles

Ocean and History Bike Tours decided to rent their bikes rather than supply their own. They received estimates from two companies.

Rocky's Cycle Center sent a table of weekly rental fees for various numbers of bikes.

# of bikes	5	10	15	20	25	30	35	40	45	50
rental fee	\$400	535	655	770	875	975	1070	1140	1180	1200

Adrian's Bike Shop sent a graph of their weekly rental fees.



A. Which bike shop has the better deal? *For 35 bikes or fewer, the best value is at Adrian's Bike Shop. For more than 35 bikes, the best deal is at Rocky's Cycle Center.*

B. How did you use the information in the table and graph to make your decision? *You could either graph both sets of data and find the point where the graphs cross. If you put both sets of data into a table, you can locate the point at which both stores sell a certain number of bikes for the same price.*

Follow - Up

- *We would NOT connect the coordinates because # of bikes cannot be broken into smaller parts.*
- *At Adrian's since 20 bikes is about \$600, then each bike is around $600 \div 20 = \$30$ each.*
- *At Rocky's if 5 bikes cost \$400, then each bike is around $400 \div 5 = \$80$ each, but the cost per bike goes down the more bikes you rent. There is probably an initial fee if you are only renting a few bikes.*

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Problem 3.2 - Finding Customers

QUESTION: *How would the price of the tour be related to the number of customers who would take the tour? The higher the price, the fewer people will take the tour. In other words, as price increases, the number of customers decreases.*

The college students did some research to see how much people would pay to go on their tour. Below is a table representing what they found.

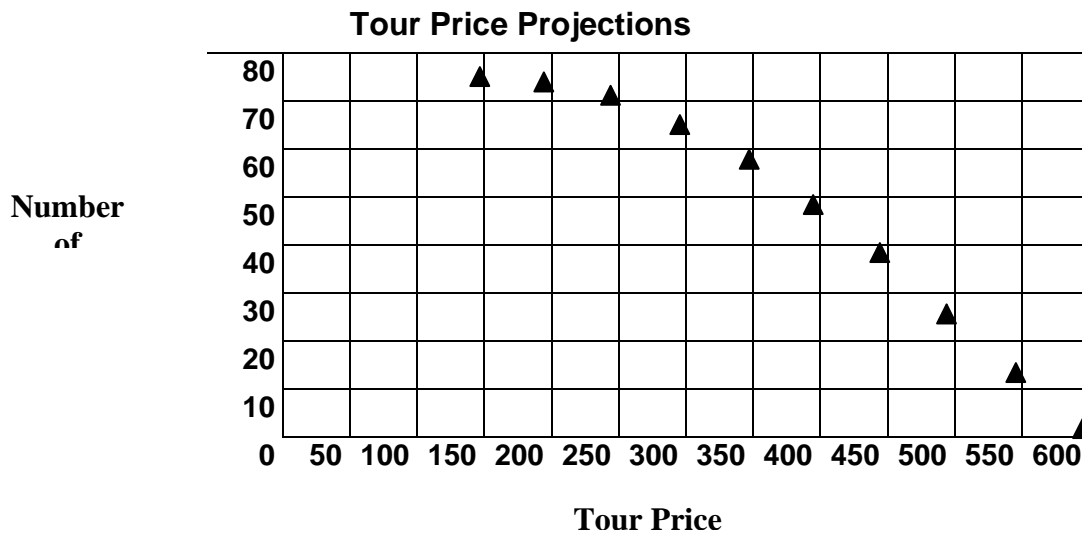
Tour Price	# who would be a customer at that price
150	76
200	74
250	71
300	65
350	59
400	49
450	38
500	26
550	14
600	0

What is the independent variable? Which axis would it go on? *Tour price: x-axis*

What is the dependent variable? Which axis would it go on? *# of customers: y-axis*

A. Why is “tour price” on the x-axis and “# of customers” on the y-axis? *The number of customers going on the tour depends on which price they charge for the tour, so price is the independent variable, and the independent variable always goes on the x-axis.*

B. Look at the graph to answer the question.



Based on the data, what price should they charge for the tour? *The number of customers starts to drop off after \$250, so we should probably charge around \$225 or \$250.*

Follow-Up

1. How does the number of potential customers change as the price increase?

As the tour price increases, the number of customers willing to take the tour begins to decrease. This is not a steady decrease, since the graph begins to curve down.

2. How do you see the change when looking at the table?

In the table the change in # of customers is shown by calculating the difference between intervals in the "# of customers" column.

3. How do you see the change when looking at the graph?

On the graph, the change is shown by how steep the lines are between the coordinates.

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Problem 3.3 - Predicting Profit

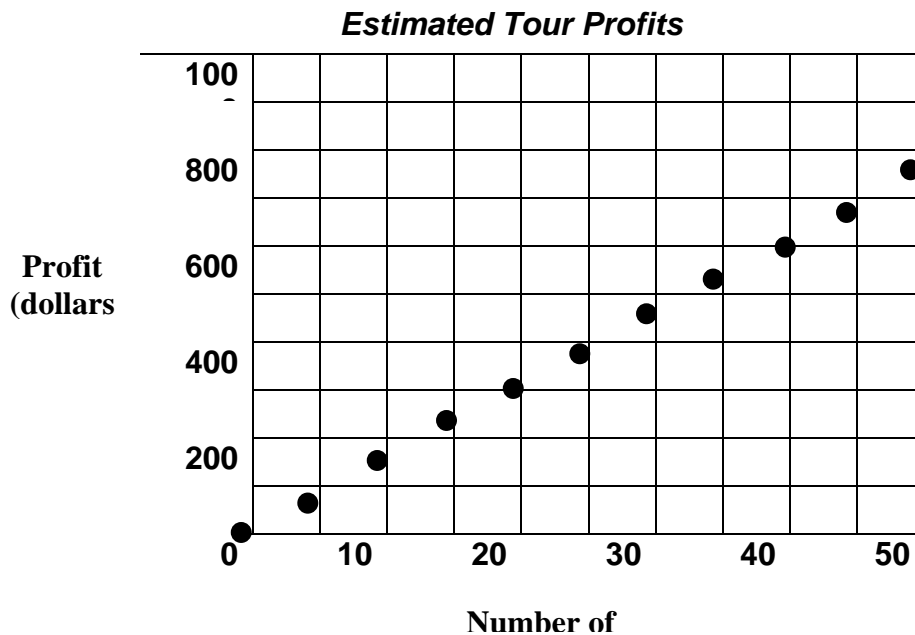
Definitions:

Profit - money remaining after expenses (cost) have been paid

Cost - actual amount of expenses

Revenue - amount of money generated (before taking out the costs)

The students decided to charge \$350 per person for the bike tour. They want to estimate their profit after taking the expenses out of the \$350. Here is a graph of how the profit depends on the number of customers going on the tour.



- A. How much profit is made if 10 customers go on the tour? 25? 40? *For 10 customers, the profit is \$200. For 25 customers, \$425. For 40 customers, \$650.*
- B. How many customers are needed for the partners to earn a \$200 profit? a \$500 profit? a \$600 profit? *In order to make \$200, they need to have 10 customers. To make \$500, they need 30 customers. To make \$600, they need 40 customers.*
- C. If the tour operators reduced their expenses but kept the price at \$350, how would this change the graph? *The graph would look similar but it would rise faster, since less expenses means more profit.*

How could we find the profit per customer, that is, how much our profit would be for each customer that takes our tour?

For 10 customers, the profit is \$200, so $200 \div 10 = \$20$ profit per customer.

This would help us to estimate our profit for 23 customers, 128 customers . . .

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Problem 3.4 - Counting Cost and Profit

Ocean and History Bike Tours decided to look carefully at the company's costs and figure the profit they would make. They considered:

- the trip will bring in about \$350 from each rider
- bike rental will cost \$30 per rider
- food and camping will cost \$125 per rider
- there will be a \$700 *one-time* fee for the van rental per tour group

# of customers	Income	Bike rental fee	Food and camp costs	Van rental	Total Cost	Profit
1	\$350	\$30	\$125	\$700	\$855	\$-505
2	700	60	250	700	1010	-310
3	1050	90	375	700	1165	-115
4	1400	120	500	700	1320	80
5	1750	150	625	700	1475	275
6	2100	180	750	700	1630	470
7	2450	210	875	700	1785	665
8	2800	240	1000	700	1940	860
9	3150	270	1125	700	2095	1050
10	3500	300	1250	700	2250	1250

Important things to look at:

- *The income increased \$350 for each customer. To find the income for any number of customers, multiply the number of riders times 350.*
- *The TOTAL cost increased by \$155 with each new person added. To find total cost, multiply the number of riders times 155 then add \$700 for the van.*
- *The profit increased more rapidly than the total cost.*
- *The least number of customers needed in order to make a profit is 4 customers.*

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Problem 4.1- Patterns and Rules

Definitions:

rule - a description of the relationship between two variables

equation - a rule that is expressed with mathematical numbers and symbols

formula - an equation that tells you how to find the value of a certain variable

Examples

The tour operator charges \$350 per customer.

RULE: *income = 350 x number of customers*

equation/formula: $I = 350n$

The circumference of a circle is pi times the diameter.

RULE: *circumference = π x diameter*

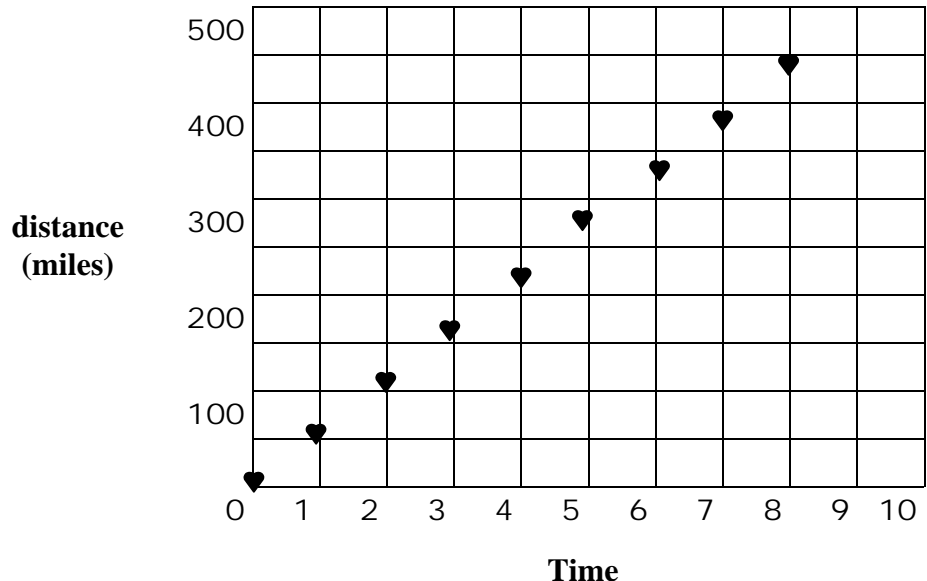
equation/formula: $C = \pi d$

If the students travel in the van at 55mph, a table and graph can show the relationship between their time and distance traveling at that speed, using:

RULE: *distance = 55 x time*

equation/formula: $d = 55t$

Time (hours)	Distance (miles)
0	0
1	55
2	110
3	165
4	220
5	275
6	330
7	385
8	440



How can you find the distance traveled after 3.25 hours by looking at the table?

look between 3 and 4 hours, and add one quarter of the difference to the distance for 3 hours

How can you find the distance traveled after 3.25 hours by looking at the graph?

on the graph, find the point on the line at 3.25 hours and go over to the y-axis to find the distance

How can you find the distance traveled after 3.25 hours by using the equation?

multiply 3.25 times 55

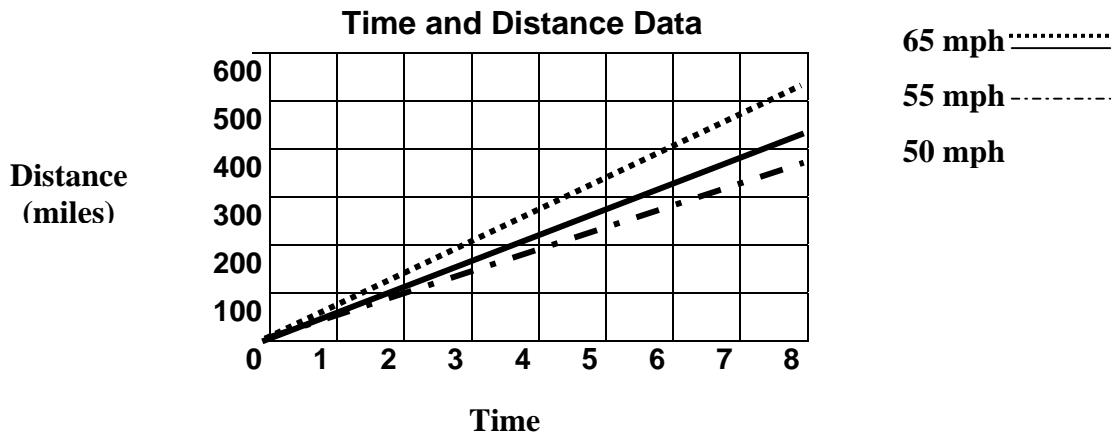
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Problem 4.2 - Changing Speeds

Suppose the group averaged different speeds while in the van. Look at what would happen to their distances if they went at a rate of 50 mph or 65 mph. Compare these with the distances going at a rate of 55 mph from yesterday.

Rate = 50 miles per hour	
Time (hours)	Distance (miles)
0	0
1	50
2	100
3	150
4	200
5	250
6	300
7	350
8	400

Rate = 65 miles per hour	
Time (hours)	Distance (miles)
0	0
1	65
2	130
3	195
4	260
5	325
6	390
7	455
8	520



How are the tables similar? How are they different? *The numbers for the distances in each table are increasing. The distances increase by different amounts.*

How are the graphs similar? How are they different? *All three graphs form a straight line. The higher the rate, the steeper the line.*

Write an equation showing the relationship between time and distance for each of the rates (50 mph and 65 mph). $d = 50t$ $d = 65t$

What would the equation $d=60t$ mean? *It means that distance is found by multiplying the time by 60. In other words, they are traveling 60 miles each hour.*

How long would it take to reach Philadelphia (310 miles) for each speed?

- going 50 mph will take 6.2 hours*
- going 60 mph will take 5.17 hours*
- going 65 mph will take 4.77 hours*

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Problem 4.3 - Equations

Look back at the table you created from problem 4.3.

A.) Write an equation for the rule to calculate each of the following the costs for any number of customers. Use n to stand for the number of customers.

- | | | |
|----------------|------------------------|---------------|
| 1. bike rental | 2. food and camp costs | 3. van rental |
| $C = 30n$ | $C = 125n$ | $C = 700$ |

B.) Write an equation for the total cost for any number of customers.

$$C = 30n + 125n + 700 \quad \text{or} \quad C = 155n + 700$$

C.) Write an equation for the profit for any number of customers.

$$P = 350n - 30n - 125n - 700 \quad \text{or} \quad P = 195n - 700$$

Follow-Up

1. Theo's father has a van he will let the students use at no charge. Which equation would represent the total cost of they used his van instead of renting one?

- a. $C=125 + 30$
- b. $C=125n + 30n$
- c. $C=155$
- d. $C=155 + n$

Answer b is the correct answer.

It's every cost except the \$700 van.

2. If the partners require customers to supply their own bikes, which of these equations would represent the total cost (assuming they are still renting the van)?

- a. $C=125n + 700$
- b. $C=125 + 700 + n$
- c. $C= 825n$
- d. $C= 350n + 125N + 700$

Answer a is the correct answer.

It's every cost except the \$30 per bike.

3. If customers must supply their own bikes, which equation will represent **Profit**?

- a. $C= 350 - (125 + 700 + n)$
- b. $C= 350n - 125n + 700$
- c. $C= 350n - (125n + 700)$
- d. $C= 350n - 125n - 700$

Answers c and d are the correct answers.

350n is their income, and we must subtract all of their costs from that amount.