How Likely Is It? - Problem 1.1 Notes
"Flipping for Breakfast"

Kalvin wants to eat COCOA BLAST cereal for breakfast every morning, but his mom wants him to eat **Health Nut Flakes**. As a compromise, Kalvin and his mom decided to flip a coin to see which cereal he will eat. If the coin lands on heads, Kalvin gets to eat COCOA BLAST, but if the coin lands on tails, he will have to eat the **Health Nut Flakes**.

Kalvin wants to predict how many days in the month of June that he will get to eat COCOA BLAST. Kalvin conducted an experiment in order to make his prediction. The results of the experiment are listed below.

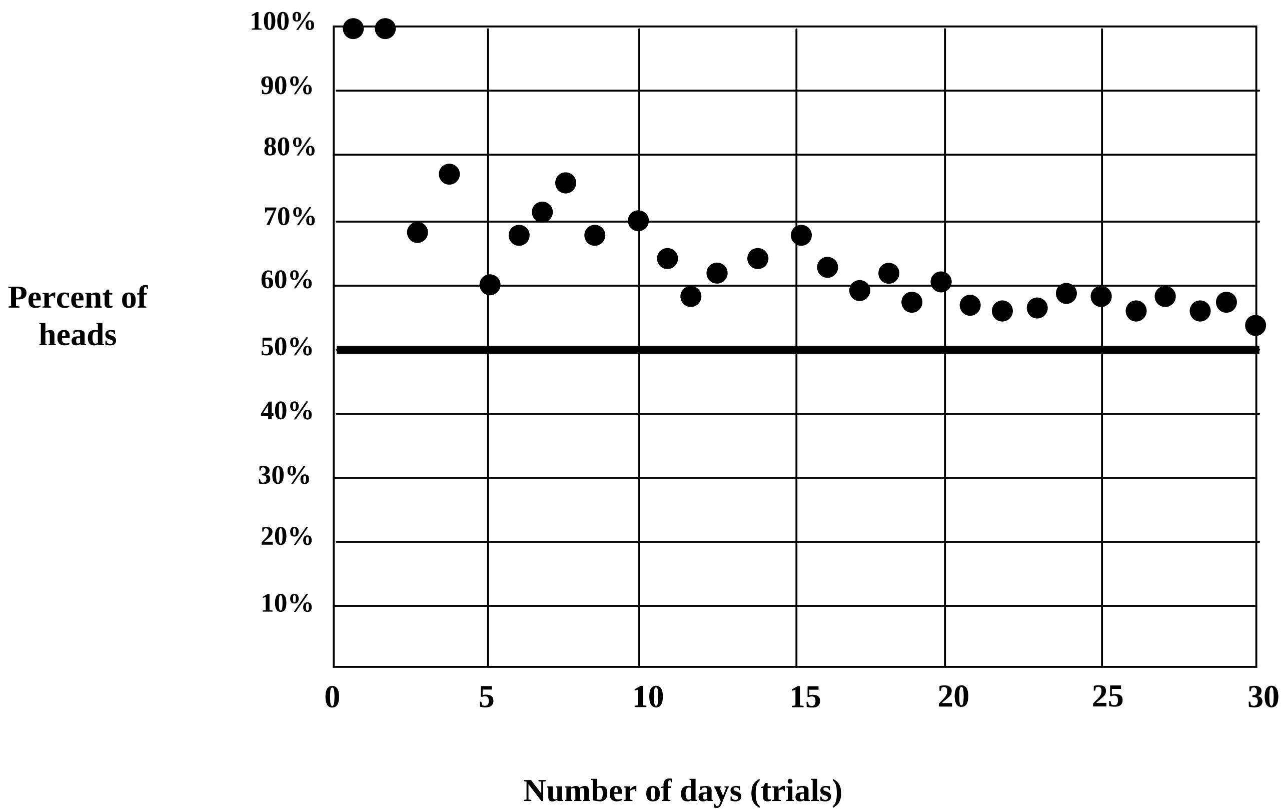
June						
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Number of heads	✓	✓		✓		✓	✓	✓		
Number of days	1	2	3	4	5	6	7	8	9	10
Percent Heads	100	100	67	75	60	67	71	75	67	60

Number of heads	✓		✓	✓	✓			✓		✓
Number of days	11	12	13	14	15	16	17	18	19	20
Percent Heads	64	58	62	64	67	63	59	61	58	60

Number of heads			✓	✓			✓		✓	
Number of days	21	22	23	24	25	26	27	28	29	30
Percent Heads	57	55	57	58	56	54	56	54	55	53

Next, Kalvin wanted to graph his information. After collecting his data, he made the coordinate graph shown below.



Follow-Up

1. a. ***Most of the time, Kalvin would flip a heads about ½ of the time.***

   b. ***As you add more and more data, you would expect the fraction of heads would get closer to ½.***

2. a. ***Based on the results from the month of June, we could expect Kalvin to eat*** COCOA BLAST ***about 15 or 16 of the 31 days in July.***

   b. ***Over a year (365 days), we would expect Kalvin to eat*** COCOA BLAST ***cereal about half the time (about 180 days).***

3. ***Kalvin's mother told him that the chances of getting a head when you flip a coin are ½. She also said that this does not guarantee that you will get a head and a tail if you flip a coin twice.***

How Likely Is It? - Problem 1.2 Notes
"Analyzing Events"

Definitions
equally likely - the same chance of getting any outcome

Kalvin found a penny by the railroad tracks that had been run over by one of the trains. He decided to use this penny to determine which cereal he would eat for breakfast. At the end of June, Kalvin's mother became suspicious. He had only eaten the Health Nut Flakes seven times. She told Kalvin that the outcomes should be equally likely when a coin is tossed. That means that you should get the same number of heads as you get tails.

Kalvins' mother gave him a list of events and asked him to decide if the outcome stated was equally likely. Here is what Kalvin discovered:

Action	Possible resulting event
A. You toss a soda can.	The can lands on its side, the can lands upside down, or the can lands right side up.
The events are not equally likely. A soda can is more likely to land on its side than on either of its ends.	
B. You roll a number cube.	1, 2, 3, 4, 5, or 6
The events are equally likely. The six sides of a fair number cube have the same chance of landing on any of the sides.	
C. You check the weather in Alaska on a December day.	It snows, it rains, or it does not rain or snow.
The events are probably not equally likely. Because of its geographic location, snow is probably more likely.	

Action	Possible resulting event
D. The Pittsburgh Steelers play a football game.	The Steelers win, the Steelers lose, or the Steelers tie.
The events are probably not equally likely, as the two teams playing are probably not exactly evenly matched. However, if the sports analyst say the game in an even match, then winning or losing might be considered equally likely.	
E. A baby is born.	The baby is a boy or the baby is a girl.
The events are roughly equally likely. Slightly more boys are born than girls.	
F. A baby is born.	The baby is right-handed or the baby is left-handed.
The events are not equally likely. More people are right-handed than left-handed.	
G. You guess on a true/false question.	The answer is right or the answer is wrong.
If you are truly guessing, the events are equally likely.	
H. You shoot a free throw.	You make the basket or you miss.
These events are probably not equally likely, since most people do not have a 50% free-throw average.	

Remember that EQUALLY LIKELY means that the outcomes of any event have the same chance of happening.

How Likely Is It? - Problem 2.1 Notes
"Tossing Marshmallows"

Definitions
none

Kalvin decided that he no longer wanted to flip a coin to decide what he will eat for breakfast. He decided that he wants to toss a marshmallow. What he needs to determine is which will give him the outcome he is looking for - a large marshmallow or a small one.

Kalvin tossed each of the marshmallows 50 times and kept track of the results in the table below.

	Lands on End	**Lands on side**
Large Marshmallow	𝍸 𝍸 𝍸 𝍸 |	𝍸 𝍸 𝍸 𝍸 𝍸 ||||
Small Marshmallow	𝍸 𝍸 𝍸 𝍸 ||||	𝍸 𝍸 𝍸 𝍸 𝍸 |

Follow-Up

1. a. For what fraction of your 50 tosses did the large marshmallow land on one of its ends? On its side?

   End is $\frac{21}{50}$ and side is $\frac{29}{50}$.

   c. If you toss a large marshmallow once each day for a year, how many times would you expect it to land on its side?

   about 210 times

2. a. For what fraction of your 50 tosses did the small marshmallow land on one of its ends? On its side?

End is $\frac{24}{50}$ and side is $\frac{26}{50}$.

c. If you toss a small marshmallow once each day for a year, how many times would you exit it to land on its side?

about 182 times

3. Suppose Kalvin uses the marshmallow you chose - large or small - to decide his cereal each morning. He tosses the marshmallow twice, and it lands on an end once and on its side once. He says, "This marshmallow isn't any better than the penny - it lands on an end 50% of the time!" How would you convince Kalvin that the marshmallow is better for him to use than a penny?

You might tell Kalvin that two trials are not enough to give a good estimate of the chances that the marshmallow will and on an end or its side. Kalvin needs to toss the marshmallow lots of times to make a good estimate.

How Likely Is It? - Problem 2.2 Notes
"Pondering Possible & Probable"

Jon and Tat Ming are playing a coin-tossing game. The rules are as follows:

two players take turns tossing three coins

★ if all 3 coins match, Jon gets 1 point
★ if only 2 coins match, Tat Ming gets a point
★ the first player to get 5 points wins

Both players have won, but Jon thinks the game is unfair because Tat Ming has won more times.

Is the game fair as long as it is possible for each player to win?

A. Is it possible for either Jon or Tat Ming to win the game?

Since it is possible for only two of the three coins or for all three coins to match, it is possible for either player to win.

B. Who is more likely to win and why?

Tat Mining is more likely to win, because the chances of two coins matching are greater than the chances of three coins matching.

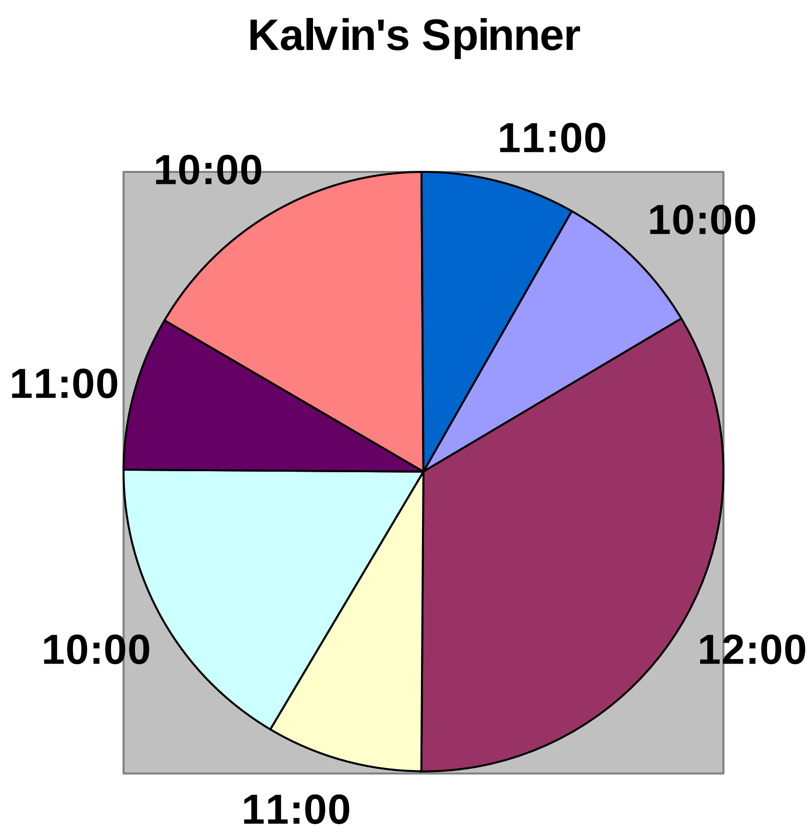
C. Is this a fair game of chance?

No, this is not a fair game of chance. Tat Ming has more chances to score, since he must match only two coins.

How Likely Is It? - Problem 3.1 Notes

"Bargaining a Better Bedtime"

Kalvin is trying to convince his father that he should get to stay up until midnight every night during the summer. In order make a more convincing argument, Kalvin made the following spinner. It contains three 10:00 sections, three 11:00 sections, and one section for 12:00 that is bigger than any of the other sections.



A. Kalvin prefers to go to bed at midnight, so he wants his spinner to land on 12:00 more often than anywhere else. Is it likely that this spinner will allow him to achieve his goal?

No; his spinner will land on 10:00 most often because the 10:00 spaces make up the largest part of the spinner.

B. Suppose Kalvin's father lets him use this spinner to determine his bedtime. What are Kalvin's chances of going to bed at 12:00?

His chance of going to bed at 12:00 are $\frac{1}{3}$ or about 33%

How Likely Is It? - Problem 4.1 Notes
"Predicting to Win"

Definitions

experimental probability - **the probability calculated based on the results of an experiment**

theoretical probability - **the probability calculated on the chance that a favorable outcome will occur**

In the previous investigations, you worked with problems involving the chance that a particular event would occur. This is known as **probability**.

In the last 5 minutes of the Gee Whiz Everyone Wins! television game show, all the members of the studio audience are called to the stage to select a block randomly from a bucket containing an unknown number of red, yellow, and blue blocks. Before drawing, each contestant is asked to predict the color of the block he or she will draw. If the guess is correct, the contestant wins a prize. After each draw, the block is put back into the bucket.

If a class of 25 students drew 15 red blocks, 6 yellow, and 4 blue blocks, we could write the following statements based on our experiment.

P(red) = $\frac{15}{25}$ **P(yellow) =** $\frac{6}{25}$ **P(blue) =** $\frac{4}{25}$

These are called the **experimental probabilities**.

If there were actually 18 blocks in the bag and there were only 9 red, 6 yellow, and 3 blue, then the probabilities would be:

P(red) = $\frac{9}{18}$ **P(yellow) =** $\frac{6}{18}$ **P(blue) =** $\frac{3}{18}$

These are called the **theoretical probabilities**.

How Likely Is It? - Problem 4.2 Notes
"Drawing More Blocks"

Your teacher put eight blocks in a bucket. All the blocks are the same size. There are three yellow, four red, and one blue.

A. When you draw a block from the bucket, are the chances equally likely that it will be yellow, red, or blue?

No, because the number of blocks is different for each different color.

B. What is the total number of blocks and how many are there of each color?

➔ **3 yellow** ➔ **4 red** ➔ **1 blue**

C. What is the **theoretical** probability of drawing each of the blocks?

P(**yellow**) = $\frac{3}{8}$ P(**red**) = $\frac{4}{8}$ P(**blue**) = $\frac{1}{8}$

Suppose we conducted an experiment where we drew blocks out of the bucket 40 times. Suppose we drew 13 yellow, 18 red, and 9 blue blocks.

D. Based on the data, what is the **experimental** probability of drawing each of the blocks?

P(**yellow**) = $\frac{13}{40}$ P(**red**) = $\frac{18}{40}$ P(**blue**) = $\frac{9}{40}$

E. Compare the theoretical probability to the experimental probability. Are the probabilities for each color close?

EP(**yellow**) = $\frac{3}{8} = \frac{15}{40}$ EP(**red**) = $\frac{4}{8} = \frac{20}{40}$ EP(**blue**) = $\frac{1}{8} = \frac{5}{40}$

TP(**yellow**) = $\frac{13}{40}$ TP(**red**) = $\frac{18}{40}$ TP(**blue**) = $\frac{9}{40}$

The probabilities are very close, but not the same. This is probably because it is unlikely the colors were drawn in the exact ratio in which they were in the bucket.

How Likely Is It? - Problem 4.3 Notes
"Winning the Bonus Prize"

All the winners from the Gee Whiz Everyone Wins! game show get an opportunity to compete for a bonus prize. Each winner draws one block from each of two bags, both of which contain one red, one yellow, and one blue block. The contestant must predict which color she or he will draw from each of the two bags. If the prediction is correct, the contestant wins a $10,000 bonus prize!

We need to conduct an experiment to determine the chances of winning for the contestants. We need to keep track of the pairs of the colors that are drawn and collect enough data to give a good estimate of the probability of drawing each pair. We need to remember that contestants must guess the color of the block they will pick from each bag. That means that we will have to count (a blue from bag 1, a red from bag 2) as a different pair from (a red from bag 1, a blue from bag 2).

A. Based on the experiment, what are a contestant's chances of winning?

Answers should be close to $\frac{1}{9}$ or 11%

B. List all the possible pairs that can be drawn from the bags. Are each of these pairs equally likely?

Each of the nine possibilities are equally likely. They are:
RR, RY, RB, YR, YY, YB, BR, BY, BB

C. What is the theoretical probability of each pair being drawn?

The theoretical probability of each pair being drawn is $\frac{1}{9}$.

D. How do the theoretical probabilities compare with the experimental probabilities?

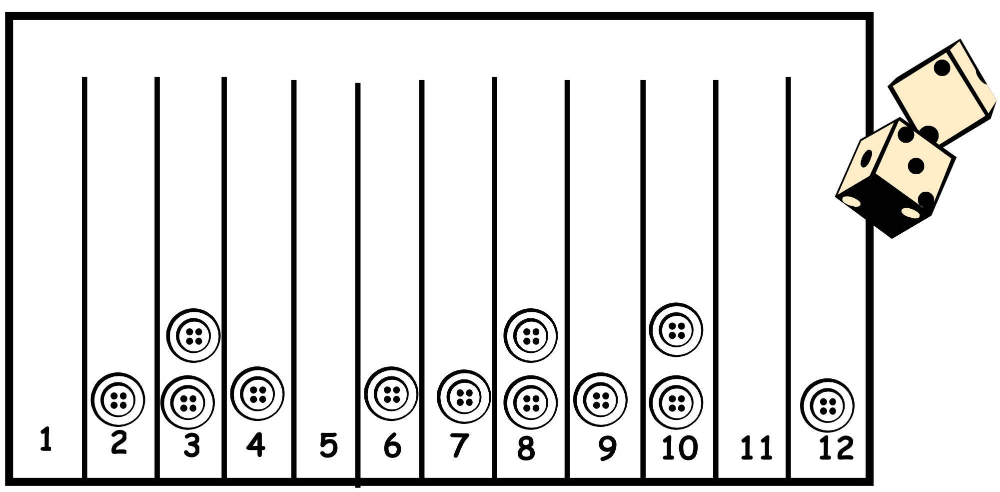
The answers may be close, but not exactly the same. If they answers are not close, it may be because not enough trials were conducted.

How Likely Is It? - Problem 5.1 Notes
"Playing Roller Derby"

In class, we played a game called Roller Derby. The game was played with two teams. Each team needed a game board, a pair of number cubes, and 12 markers. These are the rules for the game:

- ★ **Each team places its 12 markers into the columns in any way it chooses.**
- ★ **Each team rolls a number cube. The team with the highest roll goes first.**
- ★ **Teams take turns rolling the two number cubes and removing a marker from the column with the same number as the total shown on the cubes. If the column is empty, the team does not get to remove a marker.**
- ★ **The first team to remove all the markers from its board wins.**

Roller Derby Game Board



Play the game once using the gameboard on Labsheet 5.1. While you are playing, try to think of a good strategy you could use to place your markers on the board.

Summary: ***What you should realize after you have played the game is that you want to put most of your markers near the center of the board (around the 6,7, and 8) because these sums are more likely to occur than the others. You would not want to put a marker in the one column because there is no way to roll a sum of one with two dice.***

HLIT Follow-Up 5.1

a. Find a systematic way to list all the possible outcomes (number pairs) of rolling two number cubes and the sums for each of these outcomes. Analyze your list carefully before answer b-e.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. What sums are possible when you roll two cubes?

2,3,4,5,6,7,8,9,10,11, and 12

c. Which sum or sums occur most often?

The chart below shows how many times each sum occurs in the 36 possible rolls. The sums of 6, 7, and 8 occur most often. The sum of 7 occurs six times, and the sums of 6 and 8 each occur five times.

Sum	2	3	4	5	6	7	8	9	10	11	12
# of occurrences	1	2	3	4	5	6	5	4	3	2	1

d. How many ways can you get a sum of 6? And a sum of 2?

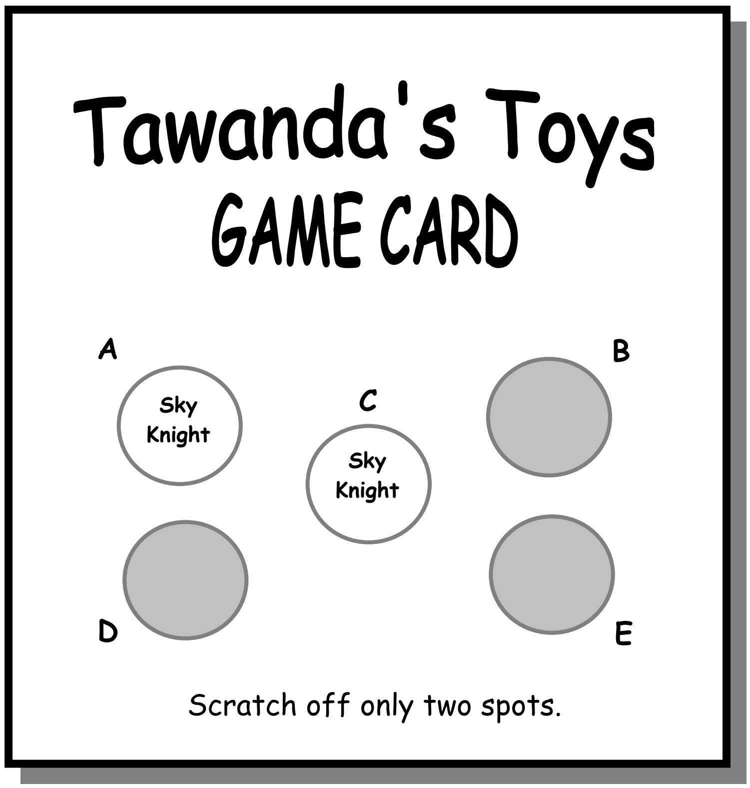
There are five ways to get a sum of 6 and one way to get a sum of 2.

e. Are all the sums equally likely?

The sums are not equally likely because you can get several of the sums in more than one way, while others you can get only one way.

How Likely Is It? - Problem 6.1 Notes
"Scratching Spots"

Tawanda's Toys is having a contest! Any customer who spends at least $10 receives a scratch-off game card. Each card has five gold spots that, when scratched, reveal the names of videos games. Exactly two spots match on each card. A customer may scratch off only two spots on a card: if the spots match, the customer wins the video game under those spots.



If you play the game once, what is your probability of winning? To answer this question, do the following two things:

1. Create a way to simulate Tawanda's contest and find the ***experimental*** probability of winning.

   Possible ways of simulating the game would be to use a spinner, five colored blocks, or pulling numbers out of a bag.

2. Analyze the different ways you can scratch off two spots, and find the ***theoretical*** probability of winning a prize with one game card.

   There are ten outcomes possible. They are: AB, AC, AD, AE, BC, BD, BE, CD, CE, and DE. Of these outcomes, only one of the pairs will be a match, so the theoretical probability of winning the game is $\frac{1}{10}$.