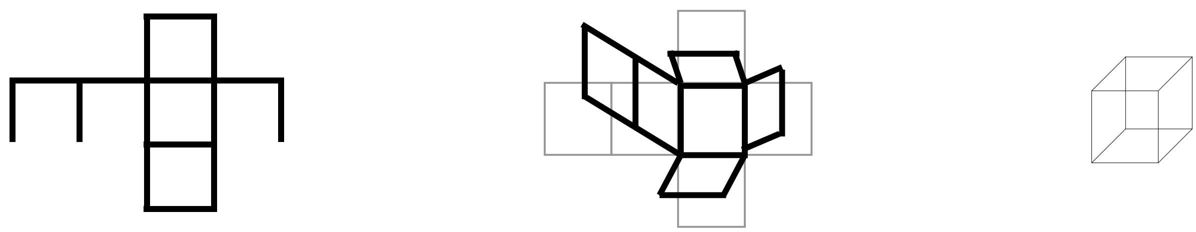
Filling & Wrapping Notes
Problem 1.1 – "Making Cubic Boxes"

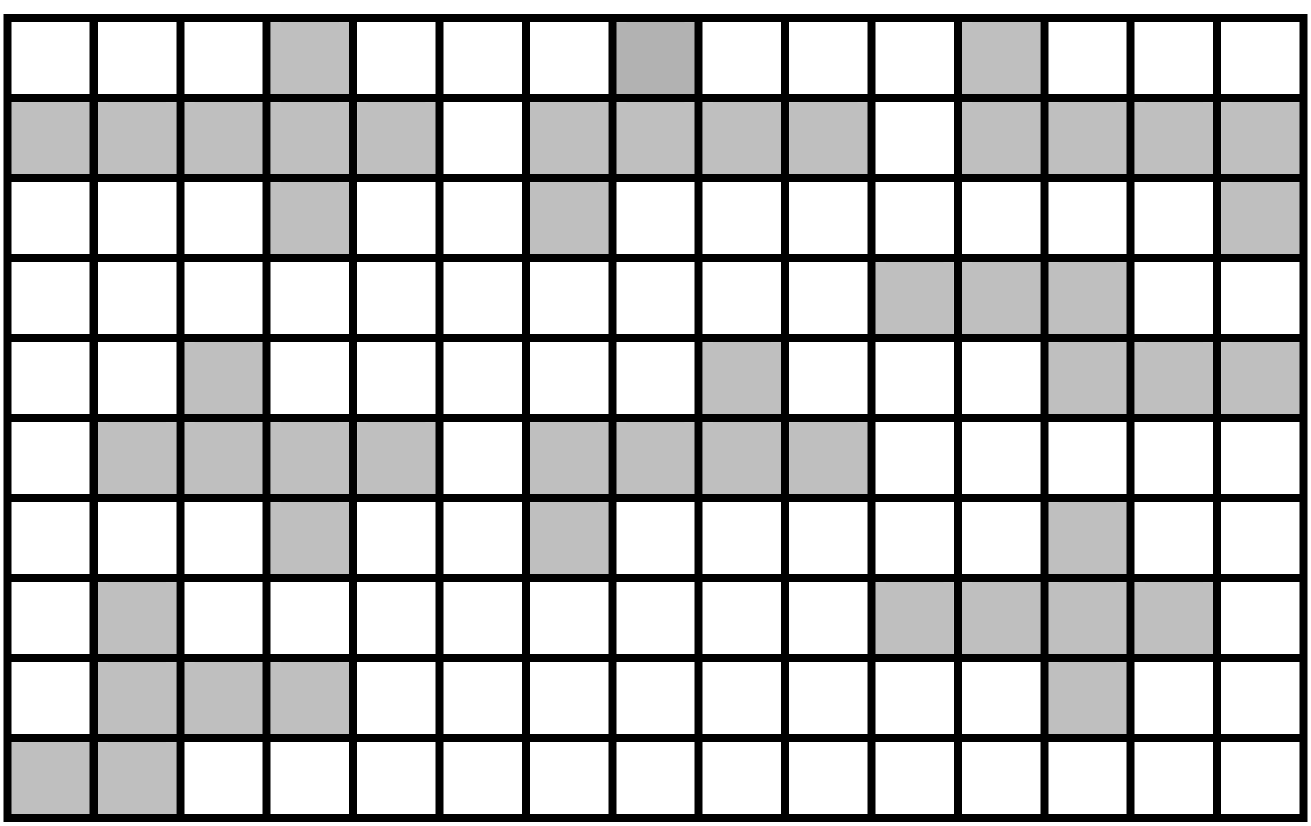
Definitions:

cube - a 3 dimensional figure with six identical square faces
unit cube - a cube with edges that are each 1 unit long
open cube - a cube with only five faces (open top)

When making boxes, we will first start with a **flat pattern** for the box. This is the sheet of paper that can be folded to created the box.



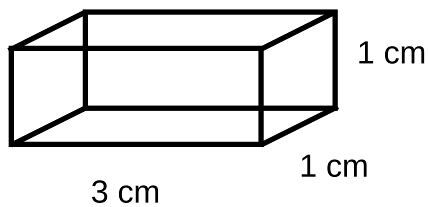
Below are some of the flat patterns that can be folded to make a unit cube. What is the area of each flat pattern? (6 cm^2)



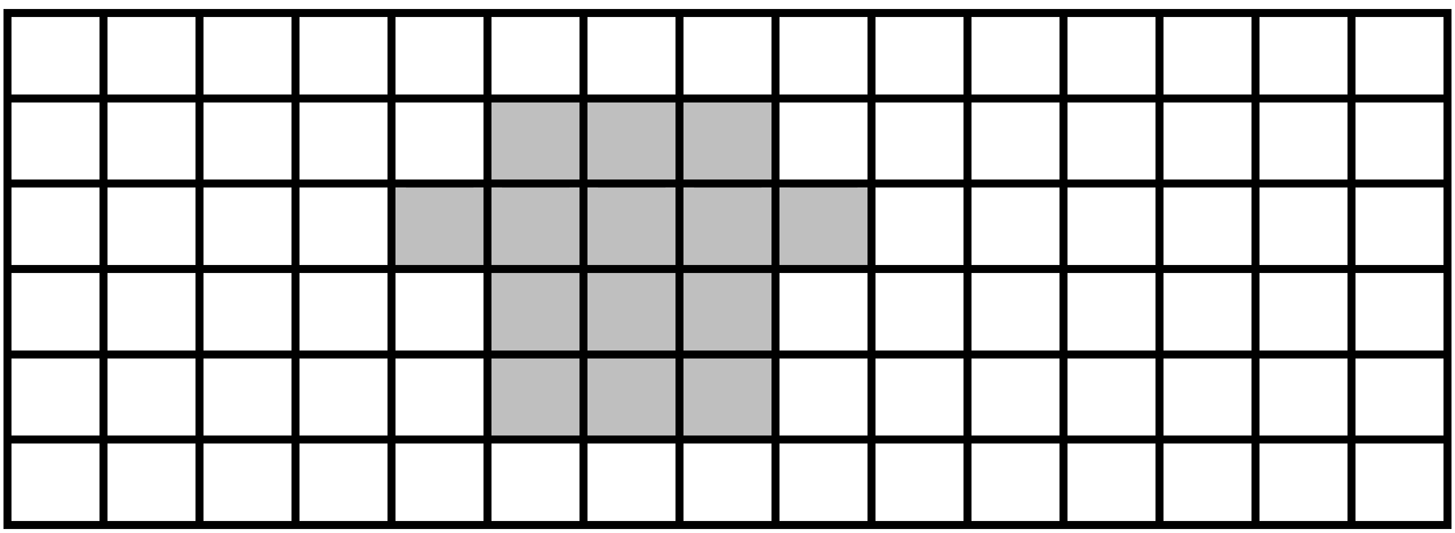
Filling & Wrapping Notes

Problem 1.2 - "Making Rectangular Boxes"

Yesterday, we only looked at cubes (every side is a square). Today we are looking at <u>rectangular boxes</u>, or prisms, where at least four of the faces are rectangles. Below is an example of a rectangular prism.



Here is one example of what the flat pattern for this box could look like:



Can you picture the pattern folded into the shape of the rectangular box?

What is the total area of the flat pattern? 14 cm^2

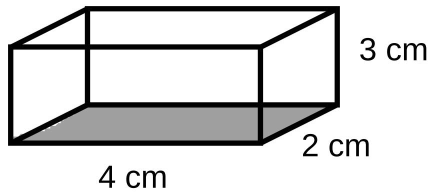
To get this answer, you could:

1) count the number of square centimeters on the flat pattern
2) find the area of each "face" and add the areas together

Filling & Wrapping Notes

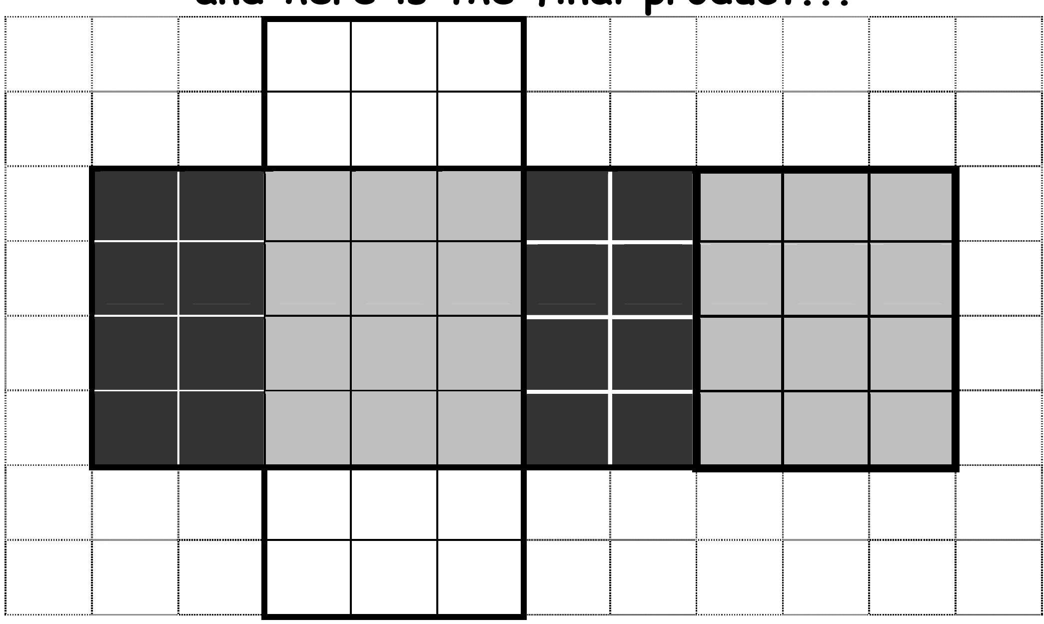
Problem 1.3 - "Making Flat Patterns for Rectangular Boxes"

Look at the cereal box below. We are going to create a flat pattern for this box.



A. Begin by listing the dimensions of the box: l = 4, w = 2, h = 3
B. Consider the base (the shaded part): we need a 2x4 rectangle
C. Next, attach the front to the base: the front is a 3x4 rectangle
D. Then, the top and back will be identical to the base and front, so repeat steps B and C.
E. Last, determine the dimensions of the left and right sides (BE CAREFUL WHERE YOU ATTACH THEM!): each side is a 2x3 rectangle

and here is the final product...



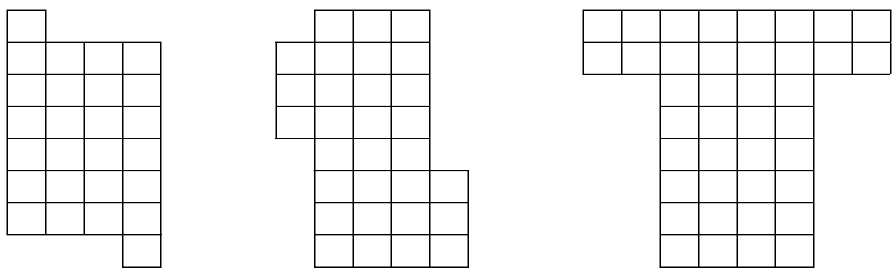
Suppose the material for this box costs $0.01 per sq. cm. How much would it cost to make this box?

It took 48 sq. cm. to make, so 48 x 0.01 = $0.48

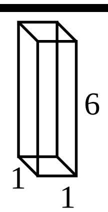
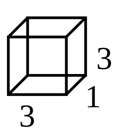
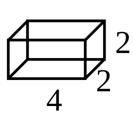
Filling & Wrapping Notes
Problem 1.4 - "Testing Flat Patterns"

Look at the flat patterns below. Each one will form a rectangular prism.

Box P **Box Q** **Box R**



A. Cut out each flat pattern and fold it to form a box - sketch the closed box.
B. Find the dimensions of each box.
C. How are the dimensions of the box related to the dimensions of each face?
D. Find the total area of all the faces of each box.
E. How many unit cubes would it take to fill each box?

BOX	*SKETCH*	*DIMENSIONS OF EACH BOX*	*DIMENSIONS OF EACH FACE*	*TOTAL AREA OF FLAT PATTERN*	*# OF UNIT CUBES TO FILL*
P		1 X 1 X 6	1 X 1 1 X 6 1 X 1 1 X 6 1 X 6 1 X 6 (combinations of 2 dim.)	1+1+6+6+6+6 **26 sq. cm.**	**6 cubic cm**
Q		1 X 3 X 3	1 X 3 1 X 3 1 X 3 3 X 3 1 X 3 3 X 3 (combinations of 2 dim.)	3+3+3+3+9+9 **30 sq. cm.**	**9 cubic cm**
R		2 X 2 X 4	2 X 2 2 X 4 2 X 2 2 X 4 2 X 4 2 X 4 (combinations of 2 dim.)	4+4+8+8+8+8 **40 sq. cm.**	**16 cubic cm**

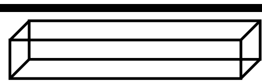
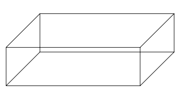
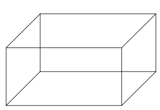
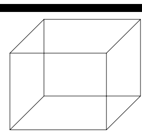
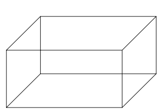
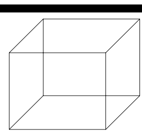
Filling & Wrapping Notes
Problem 2.1 - "Packaging Blocks"

Definitions:
surface area - the total area of all the faces of a 3-dimensional figure
volume - the number of unit cubes it takes to fill a 3-dimensional figure

The *ABC Toy Company* is planning to market a set of children's blocks. Each block is a cube with a 1-inch edge.

The company wants to arrange the 24 blocks in the shape of a rectangular prism, then package them in a box that fits them exactly.

Find all the ways 24 cubes can be arranged into a rectangular prism.

Volume	*Picture*	*L*	*W*	*H*	*Surface Area*
24 in^3	 **most surface area**	1	1	24	1•1 = 1 1•24 = 24 1•24 = 24 } = 49 49•2 = **98 in^2**
24 in^3		1	2	12	1•2 = 2 1•12 = 12 2•12 = 24 } = 38 38•2 = **76 in^2**
24 in^3		1	3	8	1•3 = 3 1•8 = 8 3•8 = 24 } = 35 35•2 = **70 in^2**
24 in^3		1	4	6	1•4 = 4 1•6 = 6 4•6 = 24 } = 34 34•2 = **68 in^2**
24 in^3		2	2	6	2•2 = 4 2•6 = 12 2•6 = 12 } = 28 28•2 = **56 in^2**
24 in^3	 **least surfac e area**	2	3	4	2•3 = 6 2•4 = 8 3•4 = 12 } = 26 26•2 = **52 in^2**

The last box will require the least amount of material to make!!!

Filling & Wrapping Notes

Problem 2.2 - "Saving Trees"

SURFACE AREA tells you how much material it takes to make a box.
VOLUME tells you how much the box can hold.

When packaging a product, we must consider which type of box will hold the most while using the least amount of material to make.

Find all the possible arrangements 8, 27, and 12 cubes. Look for a pattern to determine which has the most / least surface area.

Vol.	*dimensions*	*S.A.*
8 in^3	**1x1x8**	34 in^2
8 in^3	**1x2x4**	28 in^2
8 in^3	**2x2x2**	24 in^2

Vol.	*dimensions*	*S.A.*
12 in^3	**1x1x12**	50 in^2
12 in^3	**1x2x6**	38 in^2
12 in^3	**1x3x4**	38 in^2
12 in^3	**2x2x3**	32 in^2

Vol.	*dimensions*	*S.A.*
27 in^3	**1x1x27**	110 in^2
27 in^3	**1x3x9**	78 in^2
27 in^3	**3x3x3**	54 in^2

✷ arrangements with the **most surface area**:
1x1x8 1x1x27 1x1x12 → all are long and skinny

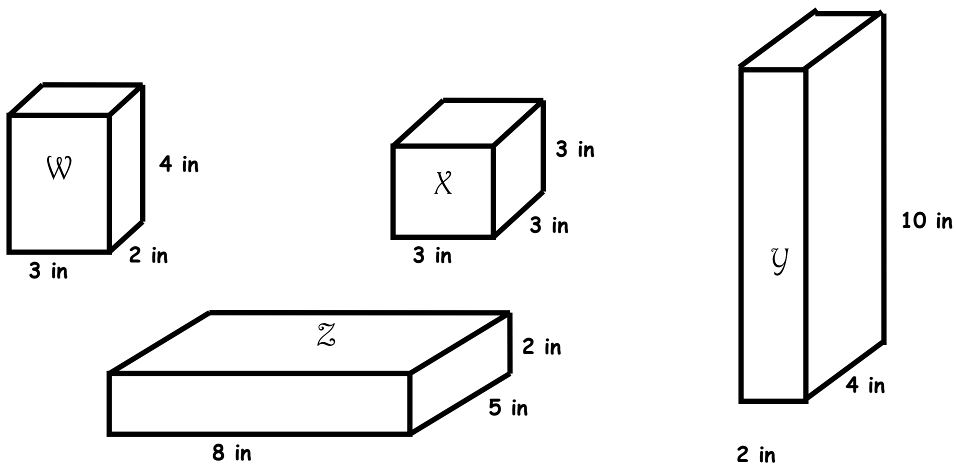
✷ arrangements with the **least surface area**:
2x2x2 3x3x3 2x2x3 → the most cube-like

Filling & Wrapping Notes

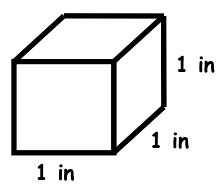
Problem 3.1 – "Volumes of Boxes"

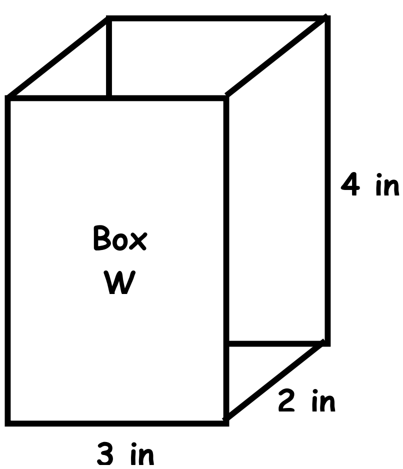
Recall that *VOLUME* tells you how much a box can hold. We measure volume in cubes (such as cm^3, in^3, ft^3, etc.)

The *ABC Toy Company* has 4 boxes to ship their alphabet blocks in. For each box, find the surfaces are and the volume.



Remember that each alphabet block is one cubic inch ($1in^3$).



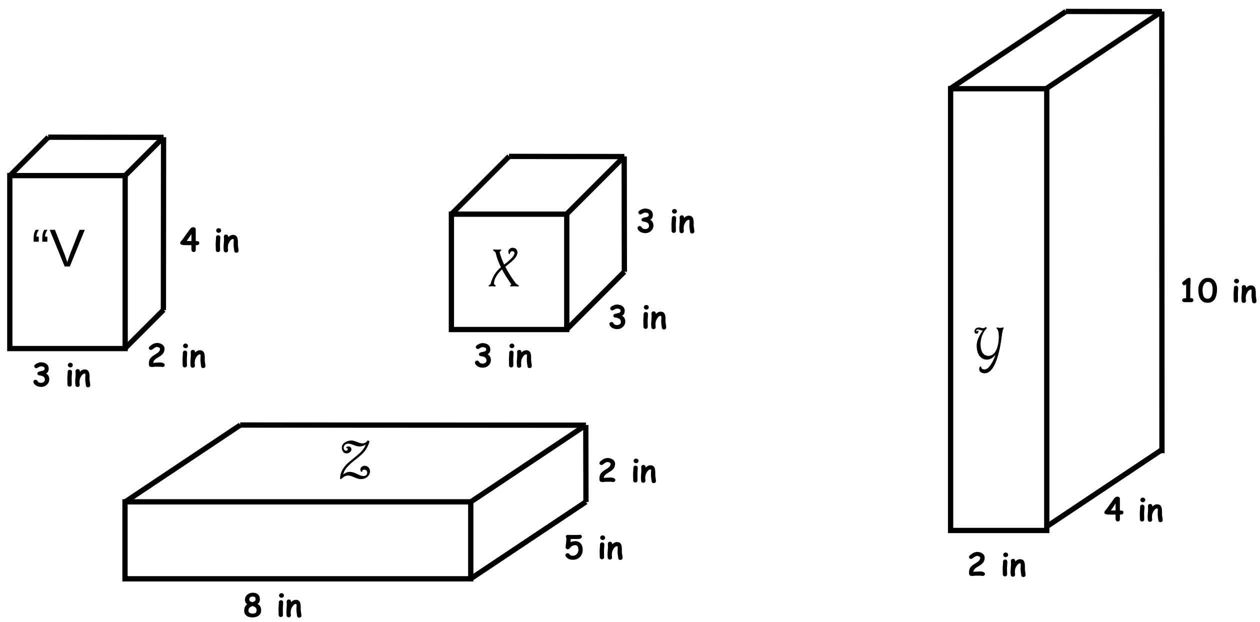


Surface Area: 3x2=6, 3x4=12, 2x4=8 } 26x2=$52in^2$

Volume: $6in^3$ will fit on the bottom layer (2x3=6), and there are 4 layers, so 6 x 4 =$24in^3$.

Filling & Wrapping Follow-Up

Problem 3.1 - "Volumes of Boxes"



- **How many unit cubes would fit in a single layer at the bottom of each box?**
- **How many *identical layers* of cubes could be stacked in each box?**
- **What is the volume of each box?**

Box	# of cubes on bottom layer	# of layers	volume
W	6	4	24 in^3
X	9	3	27 in^3
Y	8	10	80 in^3
Z	40	2	80 in^3

- **What connections do you see between the numbers in each row?**

  the # of cubes on the bottom layer x the # of layers = the volume

  in other words ... the area of the base x the height = the volume

- **Suppose box Y was laid on its side so its base was 4 inches by 10 inches and its height was 2 inches. Would this affect the volume? Explain.**

  It would not affect the volume. The box still holds the same amount.

Filling & Wrapping Notes
Problem 3.2 – "Burying Garbage"

The Trashy City of Greendale

The City of Greendale has set aside a piece of land on which to bury its garbage (this is called a waste site). They need to determine how long it will take before the site is completely filled. Here is the information we are given:

- The hole will be 500 ft wide, 200 ft long, and 75 ft deep.
- The population of Greendale is 100,000.
- A family of 4 throws away 0.4 cubic foot of garbage per day.

The solution is as follows:

First, the site will hold 500 x 200 x 75 = 7,500,000 ft^3 of garbage.

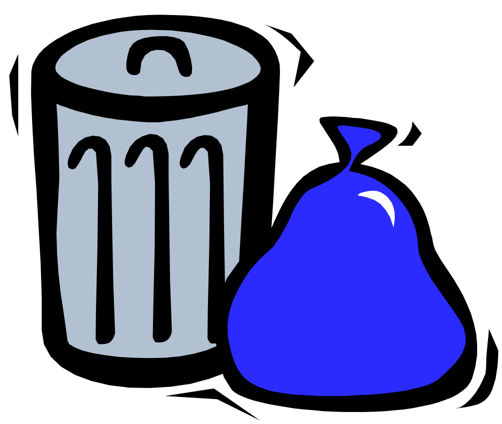
Second, each person throws away 0.4 ÷ 4 = 0.1 ft^3 of garbage per day.

Third, the whole city throws away 0.1 x 100,000 = 10,000 ft^3 of garbage per day.

Finally, we have 7,500,000 ÷ 10,000 = 750 days before the garbage site is full.

750 days ÷ 365 ≈ 2 years

In about 2 years, the waste site will be filled, and they will have to dig another.



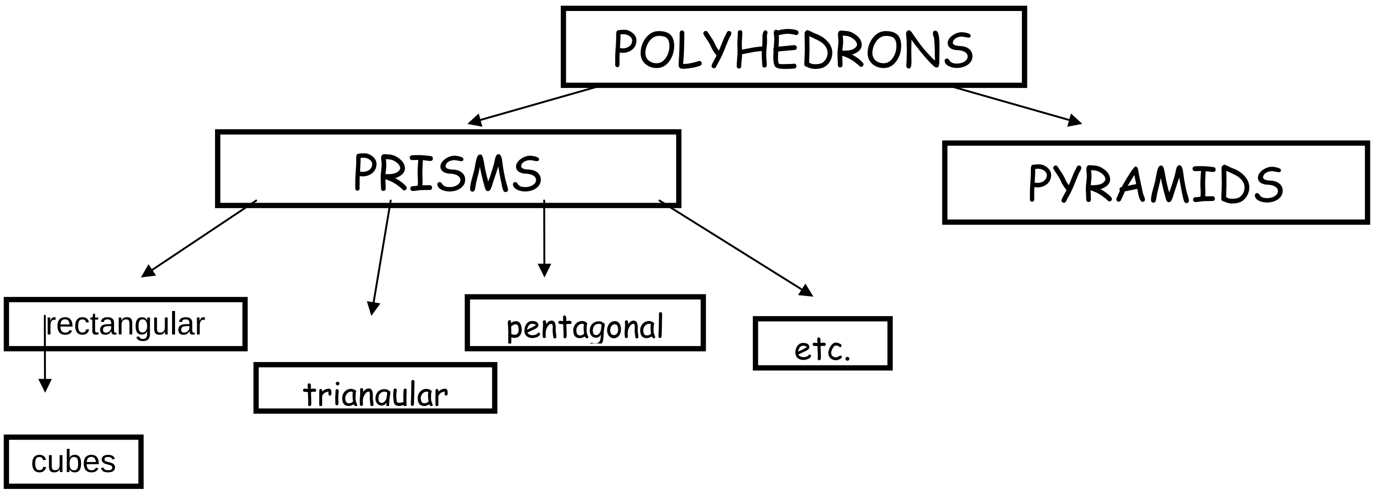
Filling & Wrapping Notes

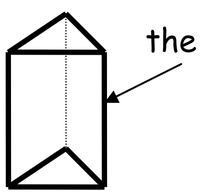
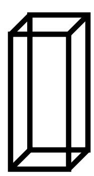
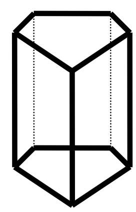
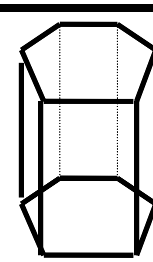
Problem 3.3 - "Fancy Boxes"

Definitions:

polyhedron - a many sided 3 dimensional figure

prism - a polyhedron with 2 congruent parallel bases



			
triangular prism	rectangular prism	pentagonal prism	hexagonal prism
5 faces (2 triangles, 3 rectangles)	6 faces (6 rectangles)	7 faces (2 pentagons, 5 rectangles)	8 faces (2 hexagons, 6 rectangles)

MUY IMPORTANTE!

/ the **HEIGHT** of the prism is always the distance between the bases (even if it is laying on its side)

/ you still find the **SURFACE AREA** by finding the area of each face and adding those areas together

/ the **VOLUME** is still the # of cubes that will fit on one layer X the # of layers, otherwise known as VOLUME = (AREA OF THE BASE) X (THE HEIGHT)

Filling & Wrapping Notes

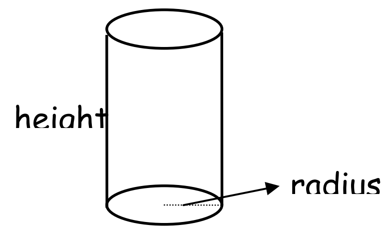
Problem 4.1 – "Filling a Cylinder"

Definitions:

Cylinder - a three dimensional figure whose top and bottom are congruent, parallel circles

Using a piece of cardstock, make a cylinder (with the top & bottom open). This is called the <u>lateral</u> **part of the cylinder.**

A. Sketch the cylinder.



B. What are the names of the dimensions of a cylinder?
Label these on your sketch from part A.

radius (or diameter) and height

C. How would you find the volume of a cylinder?
(volume ➔ the number of unit cubes it will hold)

1. find the # of cubes that fit on one layer (area of the base)
2. find the # of layers needed to fill the cylinder (height)
3. multiply <u>area of the base</u> x <u>height</u> (V = Bh)

D. Find the volume of the cylinder you made.

E. How could you use the dimensions of a cylinder to find the volume?

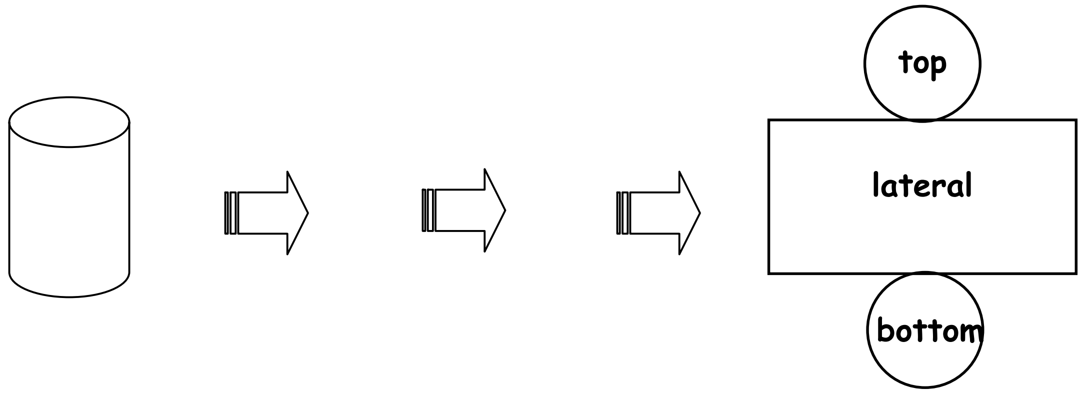
Volume = (area of the base) x (height)
Volume = (area of the circle) x (height of the cylinder)
Volume = $(\pi r^2) \times (h)$

Volume of a Cylinder = $\pi r^2 h$

Filling & Wrapping Notes

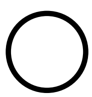
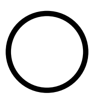
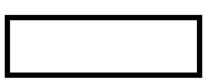
Problem 4.2 - "Flat Patterns for a Cylinder"

A. What does the flat pattern for a cylinder look like?



B. How can we find the surface area of the cylinder?

find the area of each face and add the areas together:

FACE	PICTURE	SHAPE	HOW TO FIND THE AREA
top		circle	πr^2
bottom		circle	πr^2
lateral		rectangle	**l x w** **length = the height of the cylinder (h)** **width = the circumference of the circle ($2\pi r$)** so the area of the lateral is **$2\pi rh$**

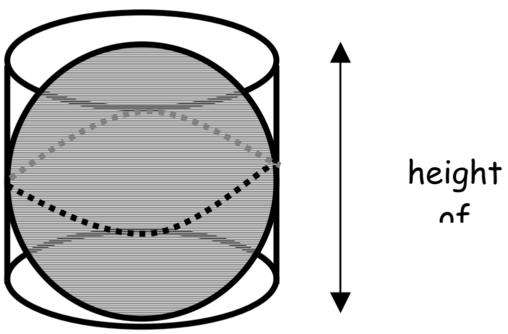
Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$

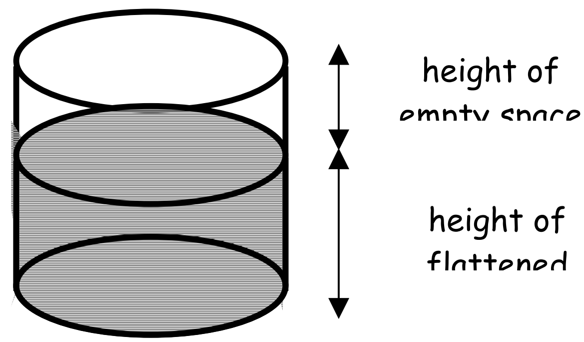
Filling & Wrapping Notes
Problem 5.1 - "Volume of a Sphere"

δ Using Playdough, make a sphere with a diameter of about 6 cm.

δ Using a cm-grid transparency, wrap the transparency around the sphere. Cut the transparency so that it forms a cylinder with the same diameter and height as the sphere.

δ Flatten the sphere so that it fits tightly in the bottom of the cylinder.





δ The flattened sphere forms a smaller cylinder inside the other cylinder.

δ Measure the height of the original cylinder. ***6 cm***

δ Measure the height of the flattened sphere. ***4 cm***

What is the relationship between the volume of the cylinder and the volume of the sphere?

$$\frac{\textbf{height of Playdough}}{\textbf{height of cylinder}} \quad \frac{4}{6} = \frac{2}{3}$$

They have the same diameter, and the Playdough is $\frac{2}{3}$ the height of the cylinder, so . . .

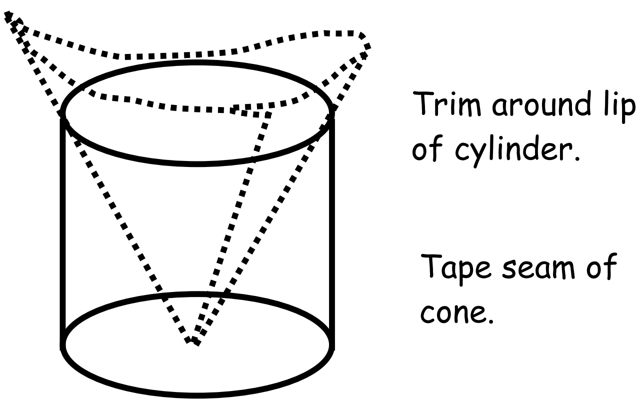
volume of the sphere = $\frac{2}{3}$ the volume of the cylinder

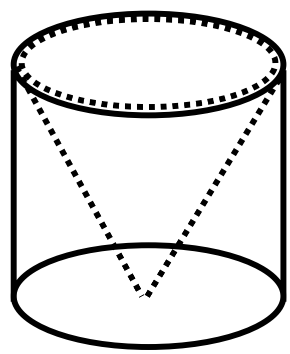
or

Volume of a Sphere = $\frac{2}{3} \pi r^2 h$

Filling & Wrapping Notes
Problem 5.2 - "Volume of a Cone"

- You will need your cylinder from Problem 5.1 and a piece of stiff paper.
- Roll the paper into a cone so that the tip touches the bottom of your cylinder.
- Tape the seam of your cone, then trim the extra paper so that your cone and cylinder have the same radius and height.





- Fill the cone to the top with rice. Gently pour the rice into the cylinder.
- Repeat this last step until the cylinder is full of rice.

A. How many cones of rice did it take you to completely fill the cylinder?

It took exactly 3 cones to fill the cylinder.

B. How is the volume of the cone related to the volume of the cylinder?

The volume of the cone was $\frac{1}{3}$ the volume of the cylinder.

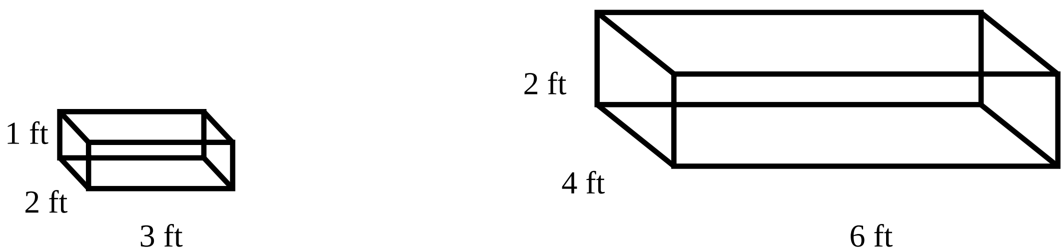
C. How can you find the volume of a cone, using the volume of a cylinder with the same radius and height?

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

Filling & Wrapping Notes
Problem 6.1 - "Scaling Boxes"

Definitions:
compost box - an open box used to turn organic waste into rich soil

Recipe for a 1-2-3 Compost Box
1. start with an open rectangular box, 1 ft high, 2 ft wide, and 3 ft long 2. fill your box with: • 10 pounds of shredded newspaper • 15 qt. of water • a few handfuls of soil • 1000 red worms 3. every day, mix your garbage with the soil in the box 4. the box will decompose about 0.5 pound of garbage per day



Mr. Greenthumbs wants to increase the size of the box so he can decompose more garbage each day. He decides to make a 2-4-6 box.

A. What was done to the dimensions of the 1-2-3 compost box to make the 2-4-6 compost box?

Each dimension became 2 times bigger (scale factor = 2).

B. How many times bigger is the volume of the 2-4-6 box?

Volume of the 1-2-3 = 6 ft^3 Volume of the 2-4-6 = 48 ft^3

The volume of the 2-4-6 box is ***8 times bigger*** than the volume of the 1-2-3 box.

C. Using your answer from part B, how will the recipe for the 2-4-6 box be different from the recipe for the 1-2-3 box?

If it's 8 times bigger we need ***8 times more*** newspaper, water, and worms.

items in the recipe	*1-2-3 compost box*	*2-4-6 compost box*
newspaper	10 pounds	10 x 8 = 80 pounds
water	15 quarts	15 x 8 = 120 quarts
worms	1,000	1000 x 8 = 8,000

D. How many pounds of garbage will the 2-4-6 box decompose daily?

The 2-4-6 box is 8 times bigger, and will be able to decompose 8 times more per day.

0.5 x 8 = *4 pounds of garbage per day*

E. Each box is made out of plywood. How much plywood is needed to make each box? (DON'T FORGET THAT THEY ARE OPEN BOXES!)

Surface area of the 1-2-3 box: 1 x 2 = 2
1 x 3 = 3
2 x 3 = 6

} 11 x 2 = 22 BUT it's an open box (the top - 6 ft^2 - is off)

22 - 6 = 16 ft^2

Surface area of the 2-4-6 box: 2 x 4 = 8
2 x 6 = 12
4 x 6 = 24

} 44 x 2 = 88 BUT it's an open box (the top - 24 ft^2 - is off)

88 - 24 = 64 ft^2

16 x 4 = 64, so we need 4 times as much plywood

SO, how does scaling up the size of a box affect the dimensions, the surface area, and the volume?

Scale factor = 2
dimensions → 2 times bigger ($2 = 2^1$)
surface area → 4 times bigger ($4 = 2^2$)
volume → 8 times bigger ($8 = 2^3$)

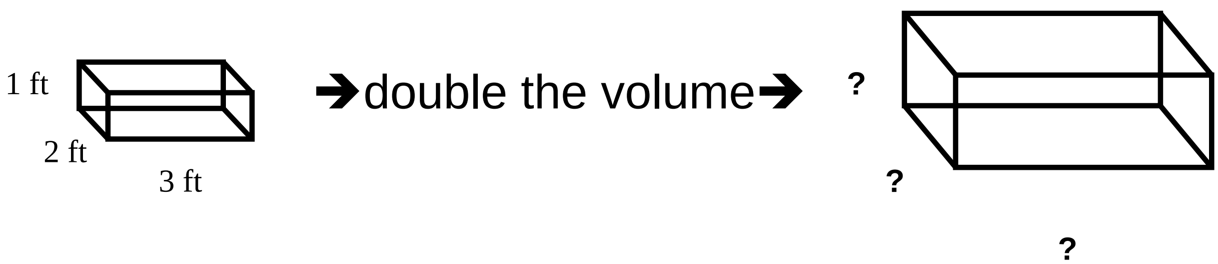
Filling & Wrapping Notes

Problem 6.2 – "Scaling Up Compost Boxes"

Recipe for a 1-2-3 Compost Box

1. start with an open rectangular box, 1 ft high, 2 ft wide, and 3 ft long
2. fill your box with:
    - 10 pounds of shredded newspaper
    - 15 qt. of water
    - a few handfuls of soil
    - 1000 red worms
3. every day, mix your garbage with the soil in the box
4. the box will decompose about 0.5 pound of garbage per day

If the original box decomposes <u>0.5 pound</u> of waster per day, how could we make a box that decomposes <u>1 pound</u> of waste per day?



A. What would be the dimensions of a compost box that decomposes 1 pound per day?

It needs to hold **<u>2 times the volume</u>**, so we need to **<u>multiply one of the dimensions times 2.</u>** The following dimensions will give us twice the volume:

2x2x3 1x4x3 1x2x6

B. How much newspaper, water, and worms would be needed?

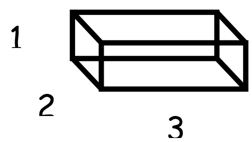
Twice the volume means we need twice as much of everything:
10 x 2 = 20 pounds of newspaper
15 x 2 = 30 quarts of water
1000 x 2 = 2,000 worms

Filling & Wrapping Notes
Problem 6.3 - "Making Similar Compost Boxes"

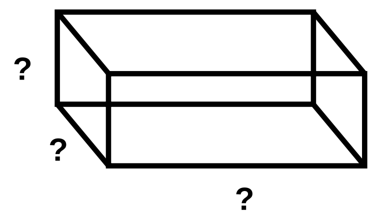
Definitions:

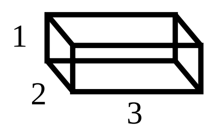
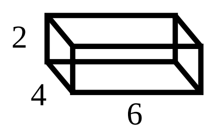
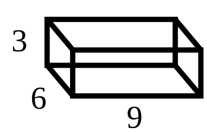
scale factor - the number each dimension of the original prism is multiplied by to create a similar prism

How do we make a SIMILAR compost box?



➔multiply by a scale factor➔



Scale Factor Used	Dimensions	Sketch	S.A. (if closed)	Volume
original box	1x2x3		22 ft^2	1x2x3 = 6 6 ft^3
2	2x4x6		88 ft^2	2x4x6 = 48 48 ft^3
3	3x6x9		198 ft^2	3x6x9 = 162 162 ft^3

old dimensions x sf = new dimensions
old surface area x sf^2 = new surface area
old volume x sf^3 = new volume