

# Data Around Us

## Problem 1.1 Notes

To make sense of the numbers in a disaster report, you may want to ask yourself two questions:

- *How accurate are the data in this report?*
- *How do the data from this disaster compare with the data from other disasters or with things I am familiar with?*

Read the following reports which describe four well-known disasters.



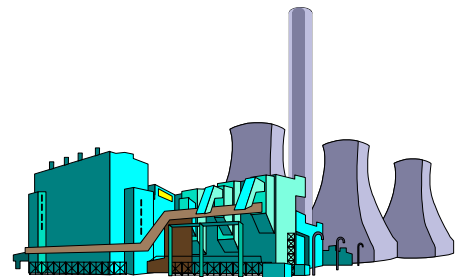
1. On October 17, 1989, as baseball's World Series was starting in San Francisco, a tremendous earthquake struck the Bay area, leaving 67 people dead, 3000 people injured, and \$10,000,000,000 in property damage. As a result of the earthquake, many major roads and bridges were closed for weeks, paralyzing traffic.

2. In the early morning of August 24, 1992, Hurricane Andrew roared across the state of Florida from Miami to the Gulf of Mexico. With a top wind speed of 164 miles per hour, it destroyed thousands of homes and businesses, caused \$20 billion in property damage, and left 15 people dead and 250,000 homeless before it moved on to Louisiana.



3. On June 9, 1991, the top of 47,985-foot Mount Pinatubo in the Philippine Islands exploded in a volcanic eruption that sent clouds of steam and ash into the atmosphere as high as 80,000 feet. The eruption poured lava as hot as 2000°F down the sides of the mountain. Pinatubo's volcanic ash fell to earth in a 60-mile radius around the mountain. The eruption caused the deaths of 700 people and destroyed 1000,000 homes and a U.S. air base.

4. On April 29, 1986, equipment in several Scandinavian countries detected dangerous levels of radioactivity in the air. The radiation was from a nuclear power plant accident near Chernobyl in the Ukraine. The accident killed at least 34 people, contaminated land for miles around, and left millions of people deeply concerned about their health.



Use the information from the reports above to answer parts A-C.

- A. Which numbers in the reports are probably very accurate, and which are probably only rough estimates?

Probably quite accurate	Probably a rough estimate
67 people dead 164 miles per hour 15 people dead 4795-foot mountain 700 deaths 34 people killed	3000 people injured \$10,000,000,000 in damage \$20 billion in damage 250,000 homeless 80,000 feet of ash and steam 2000°F lava 60-mile radius 100,000 homes destroyed contamination for miles millions of people concerned

- B. Imagine that you are a journalist writing a story about these four disasters. Write several statements you could use to compare the disasters.

*Possible answers:*

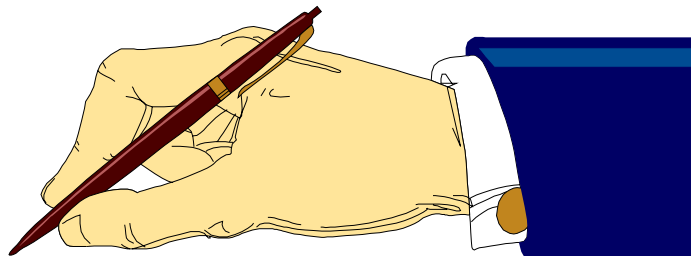
- *The damage estimate from Hurricane Andrew is \$10 billion more than, or twice, that from the World Series earthquake.*
- *The Mount Pinatubo eruption caused more than ten times the number of deaths than the World Series earthquake caused.*

*To be compared, numbers must have the same units. Measurements are commonly and more informatively compared by using differences (subtraction) or multiples, usually as a result of division.*

- C. Describe the ways you found to compare the disasters.

*Possible answer:*

- *The number of deaths can be compared by ordering the numbers or by looking at the differences.*
- *The amounts of property damage can be compared by ordering the numbers, by using subtraction, or by dividing to find a multiple.*



# Data Around Us

## Problem 2.1 Notes

Numbers are essential in reporting the size and effects of oil spills and in preparing for cleanup actions. To see how important numbers are, try reading this "censored" story about a famous oil spill.



Shortly after A on B a giant oil tanker left Valdez, Alaska, with a load of C of crude oil from the Alaskan pipeline. To avoid icebergs, the ship took a course about D out of the normal shipping channel. Unfortunately, less than E later the ship ran aground on the underwater Bligh Reef. The rocks of the reef tore a F gash in the tanker's hull, and G of crude oil spilled onto the surface of Prince William Sound. For weeks, the world watched closely as the *Exxon Valdez* oil spill became an

environmental disaster, despite extensive efforts to contain and clean up the oil. The spill gradually spread to form an oil slick, covering H of water and killing I sea otters and J birds. The cleanup engaged K of boats and workers who struggled against the cold of L water and air temperatures. The clean up cost was over M, including N for wildlife rescue alone.

Match each lettered blank in the story with the correct measurement or range of numbers from the list below:

800 square miles

2000 to 3000

40° Fahrenheit

\$2 billion

thousands

10,080,000 gallons

600-foot

150 centimeters

March 24, 1989

3 hours

\$41 million

2 miles

90,000, to 300,000

52,000,000 gallons

9:00 p.m.

- A. 9:00 p.m.
- B. March 24, 1989
- C. 52,000,000 gal
- D. 2 miles
- E. 3 hours

- F. 600-foot
- G. 10,080,000 gal
- H. 800 sq. miles
- I. 2000 to 3000
- J. 90,000 to 300,000

- K. thousands
- L. 40° Fahrenheit
- M. \$2 billion
- N. \$41 million

## Problem 2.1 Follow-Up

For parts A-N, describe what is being measured.

A. time

B. time

C. volume

D. length

E. time

F. length

G. volume

H. area

I. number of otters

J. number of birds

K. number of boats

L. temperature

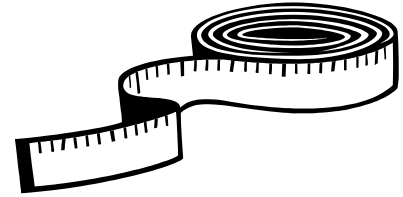
M. cost

N. cost

# Data Around Us

## Problem 2.2 Notes

The news report in Problem 2.1 gives measurements of time, length, area, volume, temperature, money, population, and so on. These measurements are given in two parts: a count and a unit of measure.



The count tells how many units of measure are being considered. For example, 10,080,000 gallons is the volume of oil spilled. The unit of measure is gallons and the count is 10,080,000.

To understand disaster reports in which large numbers are used, you need a sense of the size of different units of measure. You also need a sense of what large numbers "look like." For example, to image 10,080,000 gallons, you need a sense of "how big" a gallon is and of "how many" 10,080,000 is. In the U.S., we use two systems of measurement. The customary system includes inches, gallons, and pounds. The metric system includes meters, liters, and grams. This problem will help to refresh your memory of customary and metric units.

A-B. Think of as many units of length, area, volume, width or mass, temperature, and time as you can. Group the units by the attributes they measure.

	CUSTOMARY			METRIC		
length	<i>inch</i> <i>foot</i>	<i>yard</i> <i>mile</i>		<i>millimeter</i> <i>centimeter</i>	<i>meter</i> <i>kilometer</i>	
volume	<i>sq. inch</i> <i>sq. foot</i> <i>sq. yard</i>	<i>sq. mile</i> <i>acre</i>		<i>sq. centimeter</i> <i>sq. meter</i>	<i>sq. kilometer</i> <i>hectare</i>	
area	<i>cubic inch</i> <i>cubic foot</i> <i>cubic yard</i>	<i>cup</i> <i>pint</i> <i>quart</i> <i>gallon</i>		<i>cubic centimeter</i> <i>cubic meter</i> <i>milliliter</i>	<i>liter</i> <i>kiloliter</i>	
weight/mass	<i>ounce</i>	<i>pound</i>	<i>ton</i>	<i>milligram</i>	<i>gram</i>	<i>kilogram</i>
temperature	<i>Fahrenheit</i>			<i>Celsius</i>		
time	<i>second</i> <i>week</i>	<i>minute</i> <i>year</i>	<i>hour</i> <i>decade</i>	<i>day</i> <i>century</i>		

C. For each unit of measure, think of something familiar that is about the size of 1 unit. **Some possible benchmarks:**

- *top segment of the thumb is about 1 inch in length*
- *the diameter of a quarter is about 1 inch*
- *a paper clip has a mass of about 1 gram*
- *the area of a football field is about 1 acre*
- *a liter of water has a mass of about 1 kilogram*

## Problem 2.2 Follow-Up

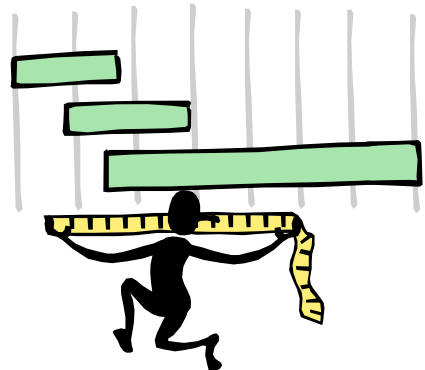
Sometimes it is convenient to know the relationship between customary and metric units of measure. The following are some common conversions:

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

$$1 \text{ gallon} = 3.785 \text{ liters}$$

$$1 \text{ pound} = 0.454 \text{ kilogram}$$

1.
  - a. Give the length of the object commonly measured in customary units, and then convert the length to metric units.  
*In the U.S., people commonly measure their height in feet and inches. A person 5 feet 2 inches tall is  $62 \times 2.54 = 157.48$  centimeters tall.*
  - b. Give the length of the object commonly measured in metric units and then convert the length to customary units.  
*In Canada, people commonly measure their height in centimeters. A person 170 centimeters tall is  $170 \div 2.54 \approx 67$  inches, or 5 feet 7 inches tall.*
2.
  - a. Give the volume of the object commonly measured in customary units, and then convert the volume to metric units.  
*Milk is commonly measured in quarts, and 1 quart  $\approx 0.946$  liter.*
  - b. Give the volume of the object commonly measured in metric units and then convert the volume to customary units.  
*In Canada, gas is commonly measured in liters, and 1 liter  $\approx 0.264$  gallon.*
3.
  - a. Give the weight of the object commonly measured in customary units, and then convert this to metric units.  
*Butter is typically measured in pounds, and 1 pound  $\approx 454$  grams.*
  - b. Give the mass of the object commonly measured in metric units and then convert this to customary units.  
*In chemistry, materials are typically measured in grams, and 1 gram  $\approx 0.035$  ounce.*
4. Find three items in your home or school that are labeled with both customary and metric units.  
*Thermometers, food items, and cleaning supplies are all common items.*



# Data Around Us

## Problem 2.3 Notes



You can imagine what a gallon of oil looks like, but can you image 10,080,000 gallons spread over 800 square miles of Prince William Sound? You have a sense of how long a foot is, but can you picture a 600-foot gash in the hull of an oil tanker? You know the value of \$1, but can you imagine the value of the \$41 million spent rescuing wildlife from the *Exxon Valdez* oil spill?

In parts A-G, facts about the *Exxon Valdez* disaster are given. Imagine you are a newspaper reporter assigned to the story. Use your own ideas for the hints given to write statements communicating each fact in a what would be easy for your readers to understand.

- A. *The Exxon Valdez spilled 10,080,000 gallons of crude oil.* Hint: An Olympic-size swimming pool holds about 500,000 gallons of water.  
**Possible answer: The oil spilled by the Exxon Valdez would fill more than 20 Olympic-size swimming pools.**
- B. *The tanker strayed about 2 miles out of the usual shipping channel.* Hint: What places in your area are about two miles apart?  
**Possible answer: The tanker was so far off course that if it were aiming at the U.S. Capitol, it would have instead hit the White House, about 2 miles away.**
- C. *The water and air temperatures during the oil spill cleanup were about 40° Fahrenheit.* Hint: When, if ever, do the water and air temperatures in your area reach these temperatures? Do you go swimming then?  
**Possible answer: The water and air temperatures during the cleanup were close to that of our estuary last January 1, when people jumped in the water to celebrate New Year's Day. They had to get out of the water immediately and dry off or risk serious injury.**
- D. *The tanker ran aground on Bligh Reef less than 3 hours after it left port.* Hint: What familiar events last about 3 hours?  
**Possible answer: In less time than it takes to play a typical baseball or football game, tanker had run aground Bligh Reef.**
- E. *The oil spill killed 90,000 to 300,000 seabirds.* Hint: Consult an almanac or atlas to find cities or towns in your state with human populations about this size.  
**Possible answer: Estimates of the number of seabirds killed by the oil spill range from 90,000, which is about the population of Gainesville, FL, to 300,000, which is about the population of Tampa, FL.**

- F. *The entire cleanup operation cost \$2,000,000,000. Hint: In the U.S., the mean annual pay for workers is about \$25,000. How many annual salaries could be paid from the cleanup cost of the oil spill?*  
**Possible answer: The cost of the cleanup was \$2,000,000,000, about equal to the average annual pay of 80,000 U.S. workers.**
- G. *The oil slick eventually covered 800 square miles of the ocean's surface. Hint: 1 mile is 5280 feet, so 1 square mile is  $5280 \times 5280 = 27,878,400$  square feet. Estimate the area of your classroom floor, then figure out how many classrooms floors it would take to cover 1 square mile of the 800 square mile oil slick.*  
**Possible answer: For classrooms measuring 25 feet by 40 feet, 1 square mile would cover  $27,878,400 \div 1000 \approx 28,000$  classrooms. Therefore, the 800-square-mile oil slick would cover roughly  $28,000 \times 800 = 22,400,000$  classrooms. Also, the oil slick would cover more than 500,000 football fields.**

## Problem 2.3 Follow-Up

The Exxon Valdez oil tanker was 987 feet long. You can get a sense of this length by imagining a "human chain."

1. Make an estimate of the arm span, in feet, of a typical seventh grader. Figure out how many seventh graders it would take to stretch out in a line as long as the *Exxon Valdez*.  
**The arm span of a typical seventh grader is about 5 feet, so it would take about 197 students standing in a line to equal the length of the tanker.**



2. Would the chain require more students than are in your class? Would it require more students than are in your school?  
**The chain would require more students than are in a typical math class, but not more students than in the school.**
3. Would this chain fit in the hall of our school? Would it fit across the football field or soccer field?  
**It is unlikely that the human chain the length of the tanker would fit in a school hallway. It would cover about three football fields and would probably be longer than the school soccer field as well.**

# Data Around Us

## Problem 3.1 Notes

To solve problems about situations like the ones described in the Data Around Us book and to discuss them others, you need to know how to write and read large numbers.

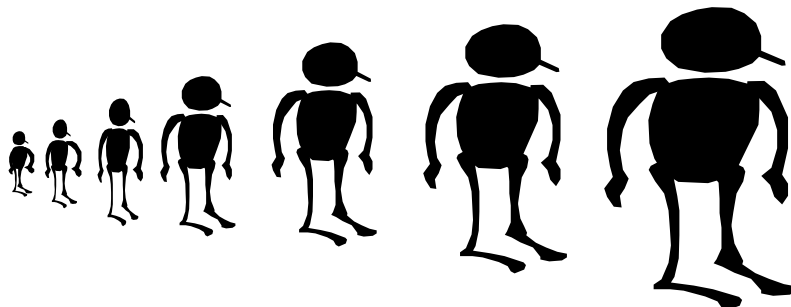
The standard notation for writing numbers involves ten digits -- 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 -- and a *place-value* system. To write and read numbers greater than 999, we group these places into clusters of three.

trillions			billions			millions			thousands			ones		
4	3	6,	5	7	2,	8	9	1,	7	5	3,	2	5	4
hundred trillions	ten trillions	trillions	hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

The number 436,572,891,753,254 is read "four hundred thirty-six trillion, five hundred seventy-two billion, eight hundred ninety-one million, seven hundred fifty-three thousand, two hundred fifty-four."

Notice that in this number the digit 2 appears in two places. In one place, it stand for two billion; in the other place is stands for two hundred.

Problem 3.1 asks you to play the Dialing Digits game. Read the directions on pg. 24 of the Data Around Us book and play the game with your parents or friends.



# Data Around Us

## Problem 3.2 Notes

You have see that reports of natural disasters often involve large numbers. Large numbers also occur frequently in population data. The table below shows 1992 populations of the 20 largest metropolitan areas in the United States. *A metropolitan area consists of a central city and smaller surrounding communities.*

Metropolitan area	Population
Atlanta	2,959,950
Boston	5,455,403
Chicago	8,239,820
Cleveland	2,859,644
Dallas	4,097,282
Detroit	5,187,171
Houston	3,731,131
Los Angeles	14,531,529
Miami	3,192,582
Minneapolis	2,538,834

Metropolitan area	Population
New York City	19,549,649
Philadelphia	5,892,937
Phoenix	2,238,480
Pittsburgh	2,394,811
St. Louis	2,492,525
San Diego	2,498,016
San Francisco	6,253,311
Seattle	2,970,328
Tampa	2,067,959
Washington, D.C.	6,727,050

Census data are given in alphabetical order, but it is often interesting and important to look at ranking by size. An increase in population can bring greater political and economic power to an area.

- A. Order the 20 metropolitan areas from most populated to least populated.  
*New York City, Los Angeles, Chicago, Washington D.C., San Francisco, Philadelphia, Boston, Detroit, Dallas, Houston, Miami, Seattle, Atlanta, Cleveland, Minneapolis, San Diego, St. Louis, Pittsburgh, Phoenix, Tampa*
- B. Describe some ways you could compare the populations of these metropolitan areas.  
*You could make comparisons by computing differences or using ratios.*
- C. Locate each of the 20 metropolitan areas on the U.S. map on Labsheet 3.2.
  1. What geographic factors seem to lead to large population centers?  
*Metropolitan areas tend to be located near coasts or along rivers and not in deserts or mountainous areas.*
  2. How do you think the locations of these large metropolitan areas affect national and state government and business decisions?
    - *Governments would consider the location of large metropolitan areas when they decide where to put new interstate highways and airports or where to concentrate on repairs and upgrades to existing structures.*
    - *Businesses would consider the location of large metropolitan areas when they decide where to build things like manufacturing plants and retail outlets.*

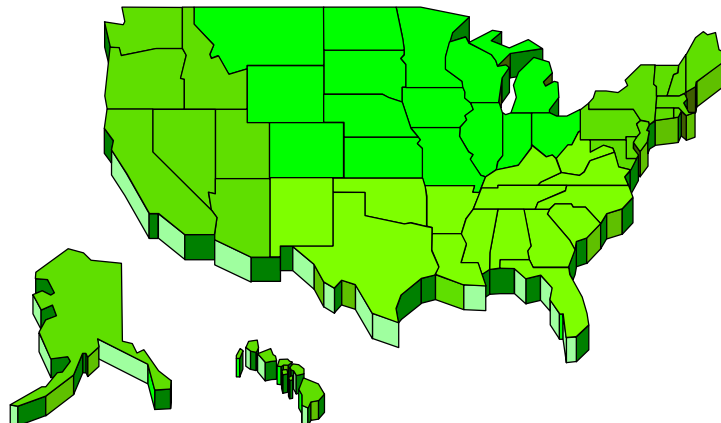
## Problem 3.2 Follow-Up

1. The table shows the population of the major city in each metropolitan area. List the cities in order from most populated to least populated.

Metropolitan area	Population of metropolitan area	Population of city
Atlanta	2,959,950	393,929
Boston	5,455,403	574,293
Chicago	8,239,820	2,783,726
Cleveland	2,859,644	505,616
Dallas	4,097,282	1,007,618
Detroit	5,187,171	1,027,974
Houston	3,731,131	1,629,902
Los Angeles	14,531,529	3,485,557
Miami	3,192,582	358,648
Minneapolis	2,538,834	368,383
New York City	19,549,649	7,322,564
Philadelphia	5,892,937	1,585,577
Phoenix	2,238,480	983,403
Pittsburgh	2,394,811	369,879
St. Louis	2,492,525	396,685
San Diego	2,498,016	1,110,554
San Francisco	6,253,311	723,959
Seattle	2,970,328	516,259
Tampa	2,067,959	280,015
Washington, D.C.	6,727,050	606,900

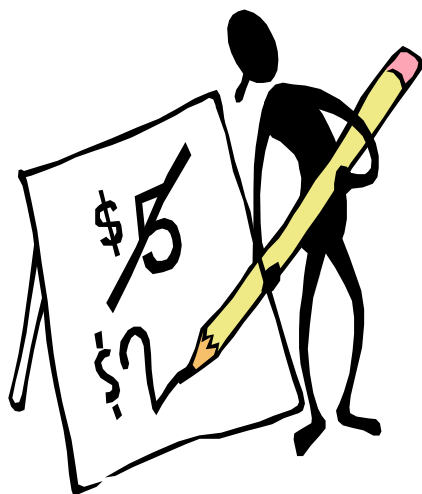
*New York City, Los Angeles, Chicago, Houston, Philadelphia, San Diego, Detroit, Dallas, Phoenix, San Francisco, Washington, D.C., Boston, Seattle, Cleveland, St. Louis, Atlanta, Pittsburgh, Minneapolis, Miami, Tampa.*

2. Compare the ranking for the cities to the ranking for the metropolitan areas. Give some possible reasons for any differences you find.
- *The three largest metropolitan areas ( NYC, LA, and Chicago) and the smallest (Tampa) are the same as the city ranking.*
  - *Except for Cleveland, the order of all the other metropolitan areas differs from the city ranking. This might be because most of the population of the three largest metropolitan areas is concentrated in the city itself, and the other metropolitan areas include many smaller suburbs.*



# Data Around Us

## Problem 3.3 Notes



The census data in Problem 3.2 are given as exact counts, but since exact populations are difficult to calculate and change daily, populations are often given as rounded figures. For example the population of the Los Angeles metropolitan area might be rounded as shown:

<u>Actual count (1992)</u>	<u>14,531,529</u>
Rounded to the nearest <i>ten million</i>	10,000,000
Rounded to the nearest <i>million</i>	15,000,000
Rounded to the nearest <i>hundred thousand</i>	14,500,000

By using rounded numbers, you can give a general idea of size without claiming exactness. And, most people find it easier to think about and compare numbers with fewer non-zero digits. When you round a number, you need to consider the situation in order to decide how accurate the rounded number should be.

In 1990, the population of the United States was reported to be 248,709,873. Here are four possible roundings of this number:

200,000,000

250,000,000

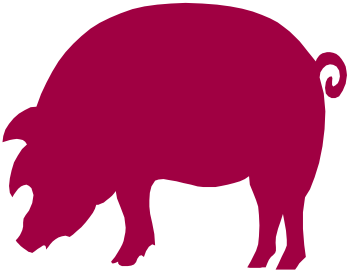
249,000,000

248,700,000

- A. The population of the world is about 5.7 billion. Which of the above roundings would you use if you wanted to compare the population of the United States with the population of the world?  
*The 200,000,000 figure is probably close enough to give a general idea of the relationship between the U.S. population and the world population.*
- B. The population of India is about 1 billion. Which rounding would you use if you wanted to compare the population of the United States with the population of India?  
*The 250,000,000 figure is probably better than the 200,000,000 figure because it shows a more accurate 4:1 ratio than the 5:1 ratio indicated by the 200,000,000 figure.*
- C. In 1980, the population of the United States was about 226,000,000. Which rounding would you use if you wanted to compare the 1990 U.S. population with this 1980 population figure?  
*The 249,000,000 figure is rounded to the nearest million, as is 226,000,000, so it is probably the best choice.*

# Data Around Us

## Problem 3.4 Notes



In areas with large populations of people or animals, finding ways to dispose of waste can be a problem. Any area must deal with the storage, treatment, disposal of garbage, and hazardous chemical wastes. Areas with industries involving great numbers of animals must also deal with the treatment and disposal of animal wastes.

The table gives data for the 15 North Carolina counties with the greatest 1993 hog populations. For each county, the table gives the 1993 hog population, the percent growth from 1983 to 1993, and the number of hogs per square mile.

County	1993 hog population	Growth from 1983	Hogs per square mile
Sampson	1,151,000	363%	1218
Duplin	1,041,000	349%	1273
Wayne	333,000	265%	603
Bladen	271,000	1178%	310
Greene	231,000	82%	870
Pitt	193,000	128%	297
Lenoir	159,000	312%	399
Johnston	129,000	61%	163
Robeson	124,000	81%	131
Onslow	115,000	261%	150
Jones	105,000	999%	223
Beaufort	103,000	51%	125
Pender	97,000	588%	111
Halifax	82,000	18%	113
Northhampton	81,000	53%	151

- A. Write at least three statements comparing the hog data for Johnston County with the hog data for Bladen County.
- *The hog population of Bladen County was more than twice the hog population of Johnston County in 1993, but this is not true in 1983.*
  - *The rate of growth between 1983 and 1993 in the hog population of Bladen County is more than 19 times what it is for Johnson County.*
  - *Bladen County has almost twice the number of hogs per square mile as does Johnston County.*
- B. Choose two different pairs of counties from the table. For each pair, write at least three statements comparing the hog data for the two counties.
- *Robeson County has 42,000 more hogs than does Halifax County.*
  - *The number of hogs in Robeson County is little more than half again the number in Halifax County, yet the number of hogs per square mile is close.*
  - *Roberson's hog population has grown at four times the rate Halifax's population has grown.*

# Data Around Us

## Problem 4.1 Notes



Large numbers appear in newspaper, radio, and television reports every day.

### How many is a million?

This problem will give you a sense of what a million "looks like."

For each part of this problem, explain how you arrived at your answer.

- A. How long does it take your heart to beat 1,000,000 times?  
*A heart that beats 70 times in one minute will beat  $70 \times 60 = 4200$  times in one hour and  $4200 \times 24 = 100,800$  times in one day. So, it would take  $1,000,000 \div 100,800 \approx 9.9$  days for the number of heartbeats to reach 1,000,000.*

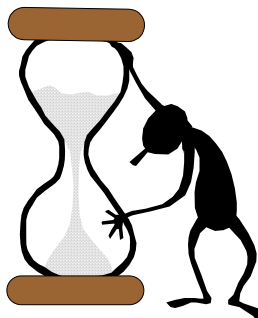
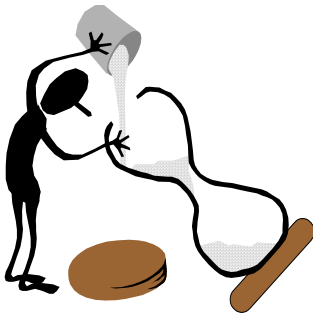
- B. Advertisements for a popular brand of chocolate chip cookie claim that there are 1000 chips in each bag of cookies. How many bags would you need to have 1,000,000 chips?  
*You would need 1000 bags to get 1,000,000 chips.*

If one bag measures 20 centimeters by 12 centimeters by 6 centimeters, would all of these bags fit in your classroom?

*The volume of 1000 bags is  $10 \times 12 \times 6 \times 1000 = 1,220,000 \text{ cm}^3$ , or  $1.44 \text{ m}^3$ , which will fit into most any classroom.*

- C. If some one is 1,000,000 hours old, what is his or her age in years?  
*As  $1,000,000 \text{ hours} \div 24 = 41,667$  days, and assuming 365 days in a year, this person is  $41,667 \div 365 \approx 114$  years old.*

- D. How many students can stand inside a square with an area of 1,000,000 square cm?  
*We could get 7 students in a square measuring 100 cm by 100 cm, or 10,000  $\text{cm}^2$ . As  $1,000,000 \div 10,000 = 100$ , 100 squares of this size would fit into a square with an area of 1,000,000  $\text{cm}^2$ . So, about  $7 \times 100 = 700$  students could stand in the larger square.*



# Data Around Us

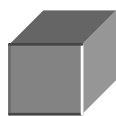
## Problem 4.2 Notes

The questions in Problem 4.1 helped you get a sense of how many a million is.

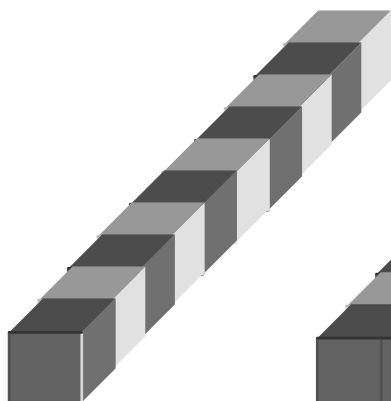
### What about a billion or a trillion?

In this problem, you will try to image a million, a billion, and a trillion as collections of unit cubes. To start, you can line up ten unit cubes to form a *long*. You put ten longs together to make a *flat*.

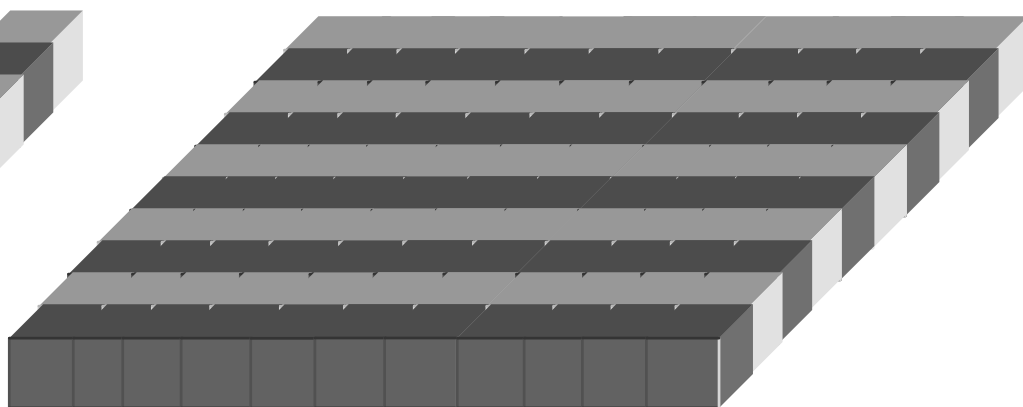
unit cube



long



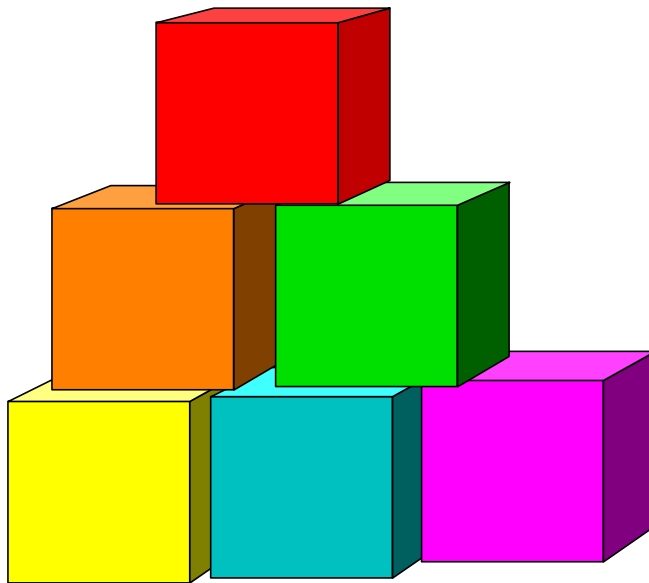
flat



- A. How many unit cubes are needed to make one flat?  
**100 (one hundred)**
- B. You can stack ten flats to make a *super cube*. How many unit cubes are needed to make a *super cube*?  
**1000 (one thousand)**
- C. You can line up ten super cubes to make a *super long*. How many unit cubes are needed to make a *super long*?  
**10,000 (ten thousand)**
- D. You can put together ten super longs to make a *super flat*. How many unit cubes are needed to make a *super flat*?  
**100,000 (one hundred thousand)**
- E. You can put together ten super flats to make a *super-duper cube*. How many unit cubes are needed to make a *super-duper cube*?  
**1,000,000 (one million) which is the same as 1000 super cubes**

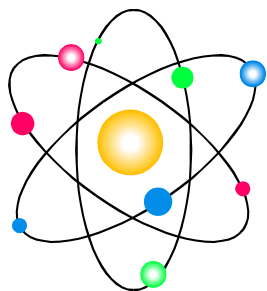
## Problem 4.2 Follow-Up

1. How many unit cubes would it take to build a *super-duper long* (made of ten super-duper cubes)? What are the dimensions of a *super-duper long*?  
***10,000,000 cubes; 1000 x 100 x 100***
2. How many unit cubes would it take to build a *super-duper flat* (made of ten super-duper longs)? What are the dimensions of a *super-duper flat*?  
***100,000,000 cubes; 1000 x 1000 x 100***
3. How many unit cubes would it take to build a *extra-super-duper cube* (made of ten super-duper flats)? What are the dimensions of a *extra-super-duper cube*?  
***1,000,000,000 cubes; 1000 x 1000 x 1000***
4. Which of the super, super-duper, or extra-super-duper arrangements contain more than a million unit cubes? More than a billion unit cubes? More than a trillion unit cubes?
  - ***The super-duper long, the super-duper flat, and the extra-super-duper cube all have more than a million cubes.***
  - ***None of the figures has more than a billion cubes, although the extra-super-duper cube has exactly a billion cubes.***
  - ***None of the figures has a trillion cubes.***



# Data Around Us

## Problem 4.3 Notes



Numbers used in scientific work are often very large. For example, there are about 33,400,000,000,000,000,000,000 molecules in 1 gram of water. There are about 25,000,000,000,000 red blood cells in a human body.

A calculator is a useful tool for working with large numbers. However, to use your calculator effectively, you need to understand the special way it handles large numbers. For example, the product of  $500,000 \times 500,000$  is 250,000,000,000. However, when you try to compute this product on your calculator, the display probably showed one of these results:

**2.5 E11**      or      **2.5 11**

Your calculator did not make a mistake, it was using a special notation. To understand your calculator's notation, start by looking at a short way to write 100,000,000,000:

$$100,000,000,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^{11}$$

In the notation  $10^{11}$ , 10 is the base and 11 is the exponent. The exponent tells you how many times the base is used as a factor.

We can use this to help find a short way to write 250,000,000,000.

$$250,000,000,000 = 2.5 \times 100,000,000,000 = 2.5 \times 10^{11}$$

The number  $2.5 \times 10^{11}$  is written in scientific notation. A number is written in scientific notation if it is expressed in the following form:

**a number greater than or equal to 1, but less than 10**

**X**

**10 raised to an exponent**

A. Write each number in standard notation.

1.  $10^{22}$       10,000,000,000,000,000,000,000
2.  $10^{13}$       10,000,000,000,000
3.  $10^{11}$       100,000,000,000
4.  $10^{10}$       10,000,000,000

B. Write each number in a short form by using an exponent.

1.  $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$        $10^{12}$
2. 1,000,000       $10^6$

C. Write each number in standard notation.

- $3.0 \times 10^9$       **3,000,000,000**
- $2.5 \times 10^{13}$       **25,000,000,000,000**
- $1.75 \times 10^{10}$       **17,500,000,000**

D. Write each number in scientific notation.

- 5,000,000       **$5 \times 10^6$**
- 18,000,000       **$1.8 \times 10^7$**
- 17,900,000,000       **$1.79 \times 10^{10}$**

E. Experiment with your calculator to figure out how to get these displays. Then, write each number in both scientific and standard notation.

- 1.7 E12** or **1.7 12**      **1,700,000,000,000;  $1.7 \times 10^{12}$**
- 1.7 E15** or **1.7 15**      **1,700,000,000,000,000;  $1.7 \times 10^{15}$**
- 2.35 E12** or **2.35 12**      **2,350,000,000,000;  $2.35 \times 10^{12}$**
- 3.698 E16** or **3.698 16**      **36,980,000,000,000,000;  $3.698 \times 10^{16}$**

## Problem 4.3 Follow-Up

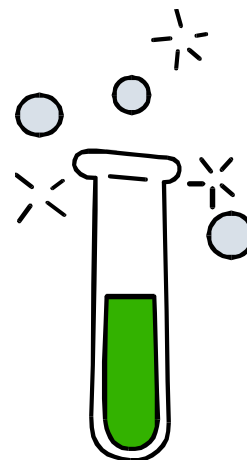
- Look back at your answers for parts C and D of problem 4.3. Compare the scientific notation for each number with the standard notation. What connections do you see between the two notations?

***The expression  $10^6$  means multiply by six factors of 10, which is also written as 1,000,000. So,  $5 \times 10^6$  means  $5 \times 1,000,000$  or 5,000,000.***

- Write each of the following as a product of the base, without using an exponent.

- $10^4$        **$10 \times 10 \times 10 \times 10$**
- $10^7$        **$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$**
- $7^3$        **$7 \times 7 \times 7$**

- Write 45,671,234,142 in scientific notation.  
 **$4.5671234142 \times 10^{10}$**



# Data Around Us

## Problem 5.1 Notes

Sometimes data are reported as totals. In Investigation 3, you looked at the total number of people in 20 major metropolitan areas. At other times, data are given as rates. In Problem 3.4, you looked at the number of hogs per square mile. In this investigation, you will continue work with data given in both forms, and you will see how you can scale rate data to find useful information.

In Problem 3.4, you looked at data for the 15 North Carolina counties that had the greatest 1993 hog populations. In this problem, you will work with the data from the top 5 counties.

County	1993 hog population	Growth from 1983	Hogs per square mile
Sampson	1,151,000	363%	1218
Duplin	1,041,000	349%	1273
Wayne	333,000	265%	603
Bladen	271,000	1178%	310
Greene	231,000	82%	870

Assume the growth of the hog populations continues at the rates given in the table for the ten years from 1993 to 2003.

- A. Predict the number of hogs in each county at the end of the year 2003. Which counties will have over a million hogs?

*Sampson:*  $1,151,000 \times 3.63 = 4,181,760 + 1,151,000 = 5,332,760 \approx 5,333,000$

*Duplin:*  $1,041,000 \times 3.49 = 3,633,090 + 1,041,000 = 4,674,090 \approx 4,674,000$

*Wayne:*  $333,000 \times 2.65 = 882,450 + 333,000 = 1,215,450 \approx 1,215,000$

*Bladen:*  $271 \times 11.78 = 3,192,380 + 271,000 = 3,463,380 \approx 3,463,000$

*Greene:*  $231,000 \times 0.82 = 189,420 + 231,000 = 420,420 \approx 420,000$

*All but Green County will have over a million hogs.*

- B. Will the ranking of these five counties be the same in 2003 as it was in 1993?

*The rankings are not the same. Bladen County moves from fourth place to third because of its enormous growth rate of 1178%.*

## Problem 5.1 Follow-Up

A square mile is about 640 acres. Find the number of hogs per acre for each county in 1993.

*Sampson:*  $1218 \div 640 = 1.90$  hogs per acre

*Duplin:*  $1273 \div 640 = 1.99$  hogs per acre

*Wayne:*  $603 \div 640 = 0.94$  hogs per acre

*Bladen:*  $310 \div 640 = 0.48$  hogs per acre

*Greene:*  $870 \div 640 = 1.36$  hogs per acre

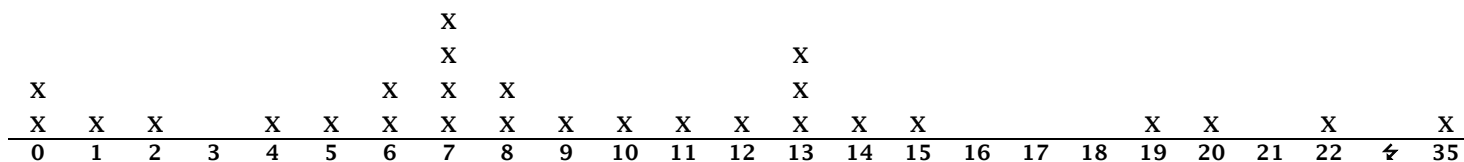
# Data Around Us

## Problem 5.2 Notes

Do you recycle your aluminum soft drink cans? You might think that recycling the small number of cans you use wouldn't make a difference. But what if everyone reasoned this way.

- A. Take a class survey, asking each student to estimate the number of soft drink cans he or she uses in a typical week. Make a line plot of the data, and find the mean and the median.

*Suppose there were 27 people in the class. One possible line plot might look like this:*



*The mean of these data is 10.1 cans per week and the median is 8 cans per week. This class decided that the median was more representative of the data.*

- B. Estimate the number of cans used by all the students in your class in one day, one week, one month, and one year.

*Using the above data where there were 27 students and an average of 8 cans per week per student, you would expect the following estimates:*

*daily: (8 cans per week ÷ 7 days in a week) x 27 students ≈ 31 cans per day*  
*weekly: 8 cans per week x 27 students = 216 cans per week*  
*monthly: 30 days in a month x 31 cans per day = 930 cans per month*  
*yearly: 52 weeks in a year x 216 cans per week = 11,232 cans per year*

- C. Estimate the number of cans used by all the students in your school in one day, one week, one month, and one year.

*If there were 467 students in the school, you would expect the following estimates:*

*daily: (8 cans per week ÷ 7 days in a week) x 467 students ≈ 534 cans per day*  
*weekly: 8 cans per week x 467 students = 3736 cans per week*  
*monthly: 30 days in a month x 534 cans per day = 16,020 cans per month*  
*yearly: 52 weeks in a year x 3736 cans per week = 194,272 cans per year*

- D. Estimate the number of cans used by all 260,000,000 Americans in one day, one week, one month, and one year.  
*To obtain estimates for all 260 million Americans, use the median of 8 cans per week to find the following estimates:*

*daily:  $(8 \div 7) \times 260$  million  $\approx 297$  million cans per day*  
*weekly:  $8 \times 260$  million  $\approx 2080$  million cans per week*  
*monthly:  $30 \times 297$  million  $\approx 8910$  million cans per month*  
*yearly:  $52 \times 2080$  million  $\approx 108,160$  million cans per year*

## Problem 5.2 Follow-Up

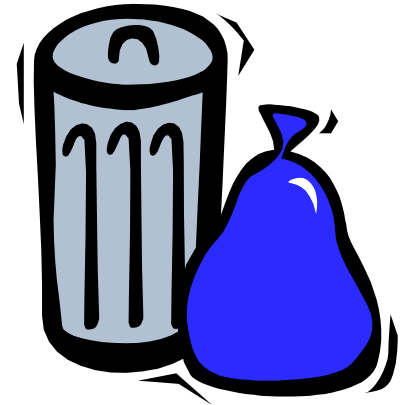
1. It takes about 20 soft drink cans to make 1 pound of recycled aluminum. There are 2000 pounds in a ton. Based on your estimates from Problem 5.2, how many tons of recycled aluminum would be produced each year if Americans recycled all their soft drink cans?  
*If you divide the result from part D by 20, you can find the number of pounds. Then, divide by 2000 to find the number of tons. So  $(108,160,000,000 \div 20) \div 2000 \approx 2,704,000$  tons of aluminum.*
2. Every ton of recycled aluminum saves 4 tons of *bauxite*, the ore from which aluminum is made. Based on your estimate from Problem 5.2, how much bauxite would be saved each year if Americans recycled all their soft drink cans?  
*Multiply the number of tons of aluminum recycled by 4, so  $2,704,000 \times 4 \approx 10,816,000$  tons of bauxite.*



# Data Around Us

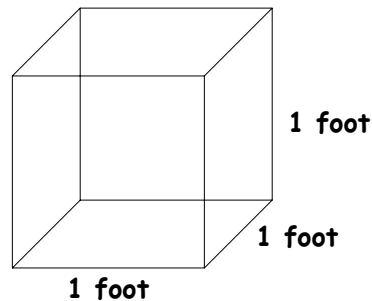
## Problem 5.4 Notes

Soft drink cans are only part of the American trash-disposal problem. Families, businesses, and factories produce many other waste materials. In 1988, the U.S. government estimated that the waste from American households and small businesses or industries, called *municipal waste*, amounts to about 4 pounds per person per day. This may not seem like much, but remember there are about 260 million Americans!



Suppose municipal waste could be compacted into cubes measuring 1 foot on each edge. Each such cube would be composed of about 50 pounds of waste.

Weight: 50 pounds



If all the municipal waste collected from American homes, businesses, and industries were pressed into 1-foot waste cubes, how many cubes would be produced in just one day?

*Americans would produce  $260,000,000 \times 4 \div 50 \approx 20,800,000$  waste cubes per day.*

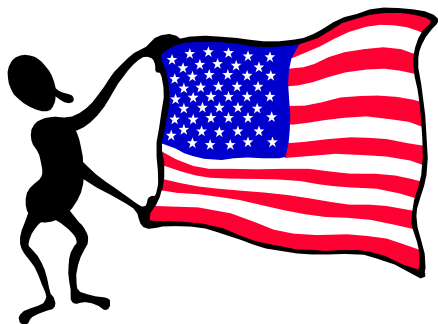
## Problem 5.4 Follow-Up

Would the waste cubes produced by Americans in one day fit in your classroom? Would the waste cubes produced by Americans in one day fit in your school?

*In a 20-foot-by-30-foot classroom, 600 cubes would fit in a layer on the floor. If the classroom were 10 feet high, 10 layers of cubes would fit in the room, for a total of  $600 \times 10 = 6000$  cubes. The daily American waste would fill roughly  $20,800,000 \div 6000 = 3740$  classrooms of this size. If the school contains 30 classrooms, plus hallways and additional rooms, the waste would fill about 100 schools.*

# Data Around Us

## Problem 6.1 Notes



The book *On an Average Day in Japan* compares life in the United States with life in Japan. The book contains lots of data about life in the two countries, and you can discover even more information by doing a little arithmetic. Note that this information was collected when the population of the United States was 250,000,000.

On an average day in Japan, about 52,055,000 aluminum cans are used. Japan's population of 123 million people recycles 34,356,000 of those cans. In the United States, which has a population of 250 million people, about 93,310,000 aluminum cans are used on average a day, and about half of them are recycled.



- A. How many aluminum cans are **not** recycled in Japan on an average day?  
 $52,055,000 - 34,356,000 = 17,699,000$  cans
- B. How many aluminum cans are **not** recycled in the United States on an average day?  
 $93,310,000 \div 2 = 46,655,000$  cans
- C. In an average week, how many aluminum cans are used in each country?  
*In an average week in Japan,  $7 \times 52,055,000 = 364,385,000$  cans are used.*  
*In an average week in the United States,  $7 \times 93,310,000 = 653,170,000$  cans are used.*

How many cans are recycled?

*In an average week in Japan,  $7 \times 34,356,000 = 240,492,000$  cans are recycled.*

*In an average week in the United States,  $7 \times 46,655,000 = 326,585,000$  cans are recycled.*

## Problem 6.1 Follow-Up

A standard aluminum can is about 12 centimeters tall. If all the cans used in Japan on an average day were stacked in a tower, how tall would the tower be in centimeters? In meters? In kilometers?

*The tower would be  $12 \times 52,055,000 = 624,660,000$  cm tall; 6,246,600 m tall; or 6246.6 km tall.*

# Data Around Us

## Problem 6.2 Notes

**There are 250 million people in the United States and 123 million people in Japan. The population of the United States is clearly greater than the population of Japan, but how can you describe how much greater?**

You could compare the populations by finding a difference. Since  $250 - 123 = 127$ , the population of the United States is about 127 million greater than the population of Japan. You could also compare the populations by figuring out how many times greater the population of the United States is. Since the population of Japan is close to 125 million, and since  $250 = 2 \times 125$ , the population of the United States is about two times the population of Japan.

Compare the data for the countries in the two ways described above, and decide which comparison better. Explain the similarities or differences.

- A. The average Japanese child spends 275 hours each year playing sports and games. The average American child spends 550 hours each year in these activities.  
***The average American child spends 275 more hours playing sports and games each year than does the average Japanese child, or two times the number of hours.***
- B. The average American has \$10,000 in savings accounts. The average Japanese has about \$40,000 saved.  
***The average Japanese has \$30,000 more in savings than does the average American, or four times the savings.***
- C. On an average day, Japanese children spend 7 hours in school, and American children spend 5 hours 25 minutes in school.  
***On an average day, Japanese children spend 1 hour 35 minutes longer in school than do American children.  
Japanese children spend about 1.3 times as many hours in school as do American children.***
- D. The average American makes 200 telephone calls each month. The average Japanese makes 45 calls each month.  
***The average American makes 155 more telephone calls per month than does the average Japanese, or about 4.4 times as many calls.***
- E. On an average day, 40% of Japanese use public transportation, while fewer than 4% of Americans do.  
***On an average day, the percentage of Japanese using public transportation is 36 percentage points greater than the percentage of Americans using public transportation.  
The percent of Japanese using public transportation on an average day is 10 times the percent of Americans using it.***

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