B&PII - Problem 1.1 notes

"Taxing Tapes"

Definitions:

percent - a way to write numbers that means "out of 100."

Percent is a special way of representing a fraction with a denominator of 100. You can think of percent as meaning "out of 100."

Sales tax is added to the cost of a purchase in order to collect money for a town, city, county, or state. A sales tax of 6% means that for every dollar an item costs, a person needs to pay an additional six hundredths of a dollar or $0.06.

For example: $1.00 + (6% of $1.00) = $1.00 + $0.06 = $1.06

Or, since $1.00 is 100 pennies:

100 pennies + (6% of 100 pennies) =
100 pennies + 6 pennies =
106 pennies - $1.06

Jill wants to buy a cassette tape that is priced at $7.50. The sales tax is 6%. What will be the total cost of the tape?

Since 6% means 6 cents for every dollar, and the price is 7.5 dollars, the tax is 6 cents x 7.5 = 45 cents, bringing the total to $7.95.

Step 1: 0.06 x 7.5 = 0.45 (calculating the tax)
Step 2: 0.45 + 7.50 = $7.95 (adding the price and the tax)

B&PII - Follow-Up 1.1 notes

You can use shortcuts to help making estimating tax easier. To find a shortcut, you can begin by examining a way to mark hundredths grids to show percents. Look on pg. 6 in your book at the hundreds boards shown.

1. A. **What is the total price for a magazine that cost $2.00 plus 6% tax?**

   $2.00 + (6% of $2.00) = $2.00 + $0.12 = $2.12

   B. **What is the total price for a book on dogs that costs $5.00 plus 6% tax?**

   $5.00 + (6% of $5.00) = $5.00 + $0.30 = $5.30

   C. **What is the total price for a comic book that costs $0.50 plus 6% tax?**

   $0.50 + (6% of $0.50) = $0.50 + $0.03 = $0.53

2. A. **What is the total price for a balloon that costs $1.00 Plus 5% tax?**

   $1.00 + (5% of $1.00) = $1.00 + $0.05 = $1.05

   B. **What is the total price for a balloon that costs $1.00 Plus 6% tax?**

   $1.00 + (6% of $1.00) = $1.00 + $0.06 = $1.06

   C. **What is the total price for a balloon that costs $1.00 Plus 7% tax?**

   $1.00 + (7% of $1.00) = $1.00 + $0.07 = $1.07

   D. **What is the total price for a balloon that costs $1.00 Plus 8% tax?**

   $1.00 + (8% of $1.00) = $1.00 + $0.08 = $1.08

3. A. **What is the total price of a pack of tennis balls that costs $5.00 plus 3% tax?**

   $5.00 + (3% of $5.00) = $5.00 + $015 = $5.15

   B. **What is the total price of a calculator that costs $19.50 plus 8% tax?**

   $19.50 + (8% of $19.50) = $19.50 + $1.56 = $21.06

4. **Kiah bought a portable cassette player. She does not remember the price, but she does know that 6% sales tax was $4.80. What was the price of the portable cassette player?** ***Since $4.80 is 480 cents and 6% is 6 cents for every dollar, then the total amount of the cassette is 480 divided by 6 which is $80.***

5. **Frank bought a new video game. The 5% sales tax was $0.75. What was the price of the game?** ***Since $0.75 is 75 cents and 5% is 5 cents for every dollar, then the total amount of the game is 75 divided by 5 which is $15.***

B&PII - Problem 1.2 notes

"Computing Tips"

At most restaurants, customers pay their server a tip for providing good service. A typical tip is 15% to 20% of the cost of the meal. Some people calculate the tip based on the cost of the meal *before* the tax is added, and others use the cost of the meal *after* the tax is added.

Look at the bill below and try to figure out what as the total bill for food and tax and how much should be left for the tip.

ξ Larry's Lunch Place β

Chicken Tenders	4.50
Iced Tea	0.80
Fresh Strawberry Pie	1.89
total	$7.19
tax (8.5%)	0.61
Total bill	$7.80

4Thanks and come again! 4

Calculating tip:

15% **10% of $7.80 is $0.78 cents and half of that is $0.39, so $0.78 + $0.39 is $1.17.**

20% **10% of $7.80 is $0.78 cents and another 10% is $0.78, so $0.78 + $0.78 is $$1.56.**

B&PII Follow-Up 1.2 Notes

1. **There are lots of strategies for determining tips. Look at how the following strategies are related.**

   A. ***For a 10% and 5% tip, the 5% is half of the 10%.***
   B. ***For a 10% and 20% tip, the 20% is twice as much as the 10%.***
   C. ***Read the following steps to find 10%, 15% and 20% of a total bill costing $17.35:***

step 1 (10%)	***10% of $17.35 is $1.74***
step 2 (15%)	***10% of $17.35 is $1.74 and half of $1.74 is $0.87 so, $1.74 + $0.87 = $2.61***
step 3 (20%)	***10% of $17.35 is $1.74 and twice that is $3.48***

2. **The sales tax in Kadisha's state is 5%. Kadisha says she computes a 15% by multiplying the tax shown on her bill by 3. For the bill of $7.93, where the total was $7.55 and the tip was $0.38, Kadisha's tip would be $0.38 x 3 = $1.14.**

   A. Kadisha's method works because the tax is 5% of the price of the food, and 15% is 3 x 5%.

   B. To calculate a 20% tip, simply multiply the tax times 4, since 4 x 5% is 20%. On Kadisha's bill, the tip would be 4 x $0.38 = $1.52.

3. **When people leave a 15% or 20% tip, the often round up to the nearest multiple of 5 or 10 cents. For example, in question 2, Kadisha might leave a tip of $1.15 rather than $1.14.**

   A. If Kadisha always rounds up, she would most likely leave $1.55 for a 20% tip.

   B. If Omar wanted to leave a $1.00 after rounding that was 20% of the bill, his meal should be anywhere from $4.76 to $5.00.

   C. If Marlene leaves a tip of $4.50 after rounding that was 15% of the meal, her meal was anywhere from $29.67 to $30.

   D. Jerome received a $2.50 tip for service. This was 20% of the bill. The bill was probably around $12.50

B&PII - Problem 1.3 notes

"Finding Bargains"

At Light Sounds Music Warehouse, CD's are regularly priced at $9.95 and tapes are regularly priced at $6.95. Every day this month, the store is offering a 10% discount on all CD's and tapes.

Special of the ψ Month ω

all CD's are 10% off

regular price $9.95

SALE - THIS MONTH ONLY!

10% off all Tapes

regular price $6.95

Joshua and Jeremy go to buy a tape and a CD. They do not have much money, so they have pooled their funds. When they get to the store, they find there is another discount plan available just for that day - if they buy three or more items, they can save 20% (instead of 10%) on each item.

A. If they buy a CD and a tape, how much money would they spend after the store adds 6% sales tax on the discounted prices?

step 1 *find the total cost before the discount: $9.95 + $6.95 = $16.90*
step 2 *compute the 10% discount: 10% x $16.90 = $1.69*
step 3 *calculate the discounted price : $16.90 - $1.69 = $15.21*
step 4 *compute the tax: 6% x $15.21 = $0.9126, which rounds to $0.91*
step 5 *compute the total cost including tax: $15.21 + $0.91*

B. Jeremy says he thinks they can buy three tapes for less money than the cost of a tape and a CD. Is he correct?

Cost of 3 tapes: *3 x $6.95 = $20.85 before the discount*
20% x $20.85 = $4.17 is the amount saved
$20.85 - $4.17 = $16.68 after the discount
$6% x $16.68 = $1.00 of tax
$16.68 + $1.00 = $17.68 is the final cost

This is more than the cost of a CD and a tape.

B&PII - Follow-Up 1.3 notes

1. **Mr. Knapp wants to take advantage of the days special to fill out his CD collection. There are 15 CD's he wants to buy.**

   a. **What is the total amount of discount he will receive?**

   15 x $9.95 = $149.25
   20% x $149.25 = $29.85 in discounts

   b. **What will the 15 CD's cost after a 6% sales tax has been added?**

   $149.25 - $29.85 = $119.40
   $119.40 x 6% = $7.16
   $119.40 + $7.16 = $126.56 total price

2. **Looking back at question 1:**

   a. ***If the discount were only 1%, Mr. Knapp would only receive $1.4925 or $1.49 (rounded) in discounts.***

   b. ***The relationship between 20% and 1% is that the 20% discount is 20 times more than the 1% discount.***

   c. ***If Mr. Knapp received a 10% discount, he would save $14.925 or $14.93 (rounded) in discounts.***

   d. ***A 10% discount is half of a 20% discount.***

   e. ***A 10% discount is 10 times as much as a 1% discount.***

   f. ***You could find a 15% discount if you know the 10% discount amount by taking half of the 10% discount and adding this amount to the 10% discount.***

   g. ***You could find a 16% discount by finding out what a 1% discount would be, then multiplying that amount by 16, since it is 16 times as much as the 1%.***

3. **You can find the percent of any number by calculating either a 1% or a 10% amount.**

B&PII - Problem 1.4 notes

"Spending Money"

Do you ever keep track of what you spend for an evening out? Are you sometimes surprised to find out you have very little money left when you get home? Danny wanted to pay more attention to where her money goes, so she decided to keep track of what she spent for an evening.

At the beginning of the evening, Danny had a twenty-dollar bill, five quarters, seven dimes, three nickels, and eight pennies.

Danny's Night Out

	Amount of money at the beginning of the evening:	20.00 1.25 0.70 0.15 + 0.08 $22.18
A.	Danny went to the Friday night school dance, which costs $2.50 to attend.	$22.18 - 2.50 $19.68
B.	After the dance, Danny and three friends bought a pizza for $6.99 and four soft drinks for $0.89 each (4 x 0.89 = $3.56). The bill for the pizza and drinks ($3.56 + 6.99 = $10.55) included a sales tax of 7% ($10.55 x 7% = $0.74).	$10.55 + $0.74 $11.29 (amount of the bill)
C.	Danny and her friends shared the cost of the pizza and drinks equally.	$11.29 ÷ 4 = $2.82 (Danny's share)
D.	On the way home, Danny stopped at a newsstand and bought a copy of *Stars and Planets* magazine for $2.50 plus 7% sales tax. (2.50 x 7% = $0.18)	$2.50 + 0.18 $2.68
	Danny's total expenses for the evening.	$2.50 2.83 +2.68 $8.01
	Amount Danny had left at the end of the evening.	$22.18 - $8.01 $14.17

B&PII - Follow-Up 1.4 notes

1. **About what *fraction* of her money did Danny *spend* during the evening?**

   Since Danny had about $22 and she spent about $8, the fraction that she spent was about $\frac{8}{22}$ ***or*** $\frac{4}{11}$.

2. **About what *fraction* of her money did Danny *have left* at the end of the evening?**

   Since Danny had about $14 left at the end of the evening, the fraction that was left was about $\frac{14}{22}$ ***or*** $\frac{7}{11}$.

3. **About what *percent* of her money did Danny *spend* during the evening?**

   $\frac{8}{22}$ ***is the same as 8 ÷ 22 which is about 0.36.***

   This is 36%.

4. **About what *percent* of her money did Danny *have left* at the end of the evening?**

   $\frac{14}{22}$ ***is the same as 14 ÷ 22 which is about 0.64.***

   This is about 64%.

B&P II - Problem 2.1 notes

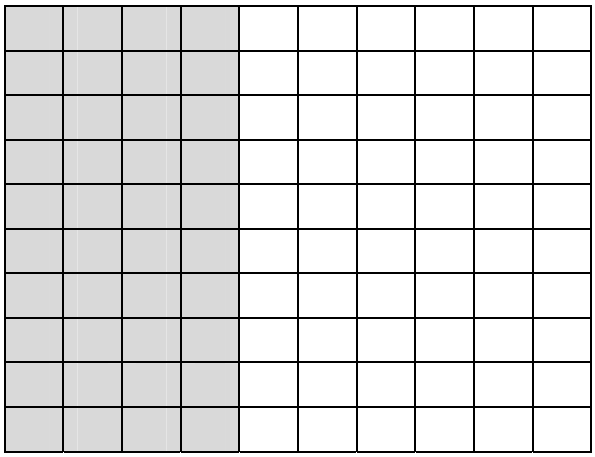
"Finding Percents"

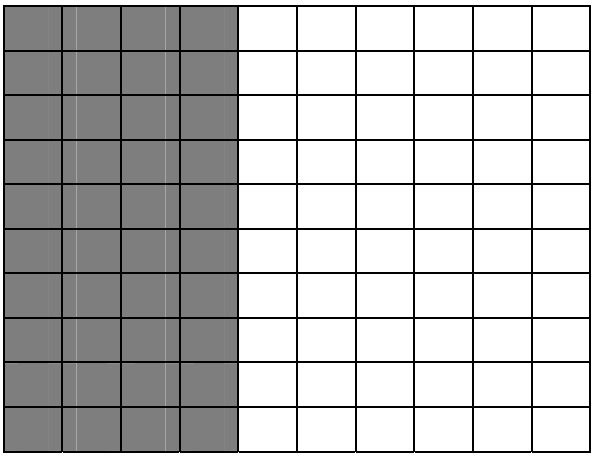
A survey asked cat owners if their cat had bad breath. There were 200 people surveyed and 80 of them said "yes." What *PERCENT* of the cat owners answered "yes?"

Remember that percent means "out of 100." There are three solution strategies that you could use to solve the problem.

Solution 1: **You could use a hundredth grid to illustrate 80 out of 200. (make sure they are evenly distributed)**

40 out of 100 are shaded in each grid.
This is 40%, so 80 out of 200 must be 40%.





Solution 2: **80 out of 200 is the fraction $\frac{80}{200}$. Create an equivalent fraction with a denominator of 100. $\frac{80}{200} = \frac{40}{100}$ and $\frac{40}{100}$ is 40%.**

Solution 3: **80 out of 200 is the fraction $\frac{80}{200}$. The fraction bar indicates division, so $80 \div 200 = 0.40$ which is 40%.**

B&P II

Problem 2.1 Follow - Up

1. If you survey 500 cat owners, about how many would you expect to say their cats have bad breath?

40% means 40 out of every 100.

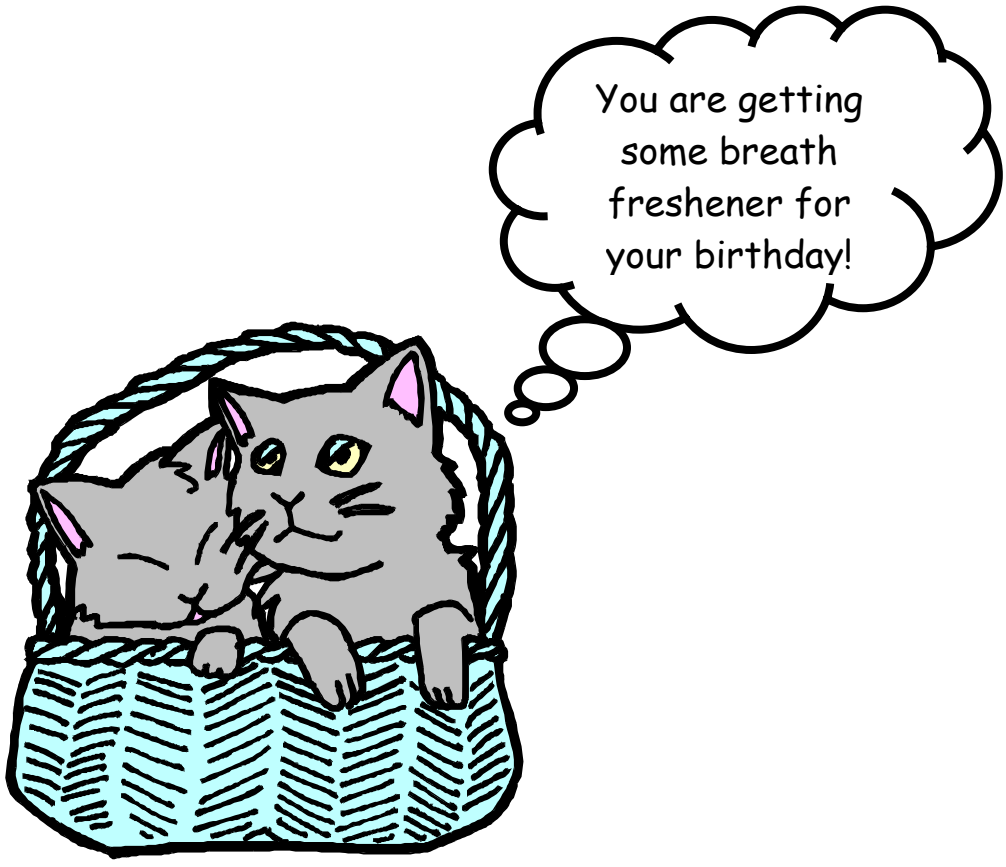
500 is 5 groups of 100.

So, 40% of 500 is like 40 cat owners x 5 groups of 100 which is 200 cat owners.

2. If you survey 75 cat owners, about how many would you expect to say their cats have bad breath?

40% means $\frac{40}{100}$, *so* $\frac{40}{100} = \frac{?}{75}$ *Since* $\frac{40}{100} = \frac{10}{25}$ *and* $\frac{10}{25} = \frac{30}{75}$,

then 30 out of 75 would say their cats had bad breath.



B&PII - Problem 2.2 notes

"Finding a General Strategy"

Sometimes there is a strategy that can be used to solve many problems. Look at the following and see if you begin to recognize a strategy.

A. **If 80 out of 400 cat owners surveyed said their cats have bad breath, what percent of the cat owners is this?** *Compare* $\frac{80}{400}$ *to* $\frac{20}{100}$, *so the answer is 20%. This percent is less than the percent represented by* $\frac{80}{200}$.

B. **If 120 out of 300 seventh graders surveyed said math is their favorite subject, what percent of these seventh graders is this?**

Compare $\frac{120}{300}$ *to* $\frac{40}{100}$, *so the answer is 40%.*

C. **If 30 out of 50 adults surveyed said they enjoyed their jobs, what percent of these adults is this?**

Compare $\frac{30}{50}$ *to* $\frac{60}{100}$, *so the answer is 60%.*

D. **If 34 out of 125 sixth graders surveyed said they would like to try hang gliding, what percent of these sixth graders is this?**

34 out of 125 is the fraction $\frac{34}{125}$. *The fraction bar indicates division, so* $34 \div 125 = 0.272$ *which is 27.2%.*

E. **If 5 out of 73 middle -school students said they look forward to fire drills, what percent of these middle-school students is this?**

5 out of 73 is the fraction $\frac{5}{73}$. *The fraction bar indicates division, so* $5 \div 73 = 0.06849...$ *which is about 6.8% or 7%.*

F. **Write an explanation for how to solve these kinds of problems.**

You can solve these problems by finding equivalent fractions and changing them to a percent or by dividing and converting the decimal answer to a percent.

B&P II Problem 2.2

Follow-Up

1. How would you find the fraction of people surveyed that answered in a given way and how does finding a fraction help you find a percent?

The total number of people surveyed represents the `whole` *(the denominator of the fraction). The number of people who answer yes to a particular question is the* `part` *(the numerator of the fraction). Once you write a fraction, you can find an equivalent fraction with a denominator of 100, which can easily be written as a percent.*

2. a) A pet store sells a new digestive mouthwash for cats. To promote the new product, the store is offering $0.50 off the regular price of $2.00 for an 8-ounce bottle. Find the percent of the discount.

The whole in the case is $2.00 and the part is $0.50. Thus, the faction representing the discount is $\frac{0.50}{2.00}$. *This fraction is equivalent to* $\frac{50}{200} = \frac{25}{100}$, *which can be written as 25%.*

b) Change the dollar amounts in part a to numbers of pennies. Now find the percent discount on the mouthwash. How do your answers compare?

You can form the fraction $\frac{50}{200}$, *which is* $\frac{1}{4}$ *or 0.25 or 25%. The two answers are the same.*

B&PII - Problem 2.3 notes

"Clipping Coupons"

The newspaper has many different types of coupons. Sometimes you want to figure out how much money you save and what percent discount is being offered.

Look at the coupon below and find the percent discount being offered.

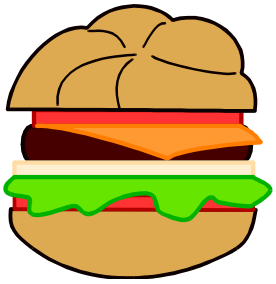
> **75 cents OFF**
>
> ***Super Deal Combo***
>
> **Any Sandwich, Large Order of French Fries, and Large Soft Drink**
>
> **Regular price** $3.00
> ***With coupon*** $2.25

Solution A:

Think of $3 as 300 pennies or $1 as 100 pennies. Since there are three groups of 100, and $\frac{75}{300}$ *is equivalent to* $\frac{25}{100}$*, the answer is a 25% discount.*

Solution B:

If 75 cents out of 3 dollars can be written as the fraction $\frac{75}{300}$*, and a fraction is a division problem, then 75 ÷ 300 is 0.25 which is a 25% discount.*



B&P II Problem 2.3

Follow-Up

1. **Estimate the percent discount you would get from a coupon that offers a regular price movie ticket of $4.50, but an early-bird ticket for $4.00.**

   Solution 1: *about 10%, since 10% of $4.50 is $0.45.*

   Solution 2: *about 11%, since* $\frac{50}{450}$ *= 50÷450 which is 0.1111.....*

2. **Estimate the percent discount you would get from a coupon that offers a set of binoculars on sale for $17.70. The regular price is $29.50.**

   Solution 1: *This amount of reduction is about $12.00, and since 10% of $29.50 is about $3.00, and 4 x $3.00 - $12.00, the percent discount is about 4 x 10% or 40%.*

   Solution 2: *$29.50-$17.70 = $11.80, and* $\frac{11.80}{29.50}$ *= 0.4 = 40%*

3. **The discount on a skateboard is $24.75, which is 25% of the original cost. What was the original cost?**

   25% is the same as $\frac{1}{4}$*, and if* $\frac{1}{4}$ *of the amount is $24.75, then the whole must be $24.75 x 4 = $99.00.*

4. **The regular price for the sneakers that Kelly wants is $68.98. The sneakers are on sale for 20% off. A sales tax of 6% will be computed on the sale price. How much will Kelly pay for the shoes?**

   Step 1: *Find the discount. $68.98 x 20% = $13.796, which rounds to $13.80*
   Step 2: *Find the sales price. $68.98 - 13.80 = $55.18*
   Step 3: *Calculate the tax. $55.18 x 6% = $3.3108, which rounds to $3.31*
   Step 4: *Find the total cost. $55.18 + 3.31 - $58.49*

   Answer: *$58.49 or $58.50*

B&PII - Problem 2.4 notes

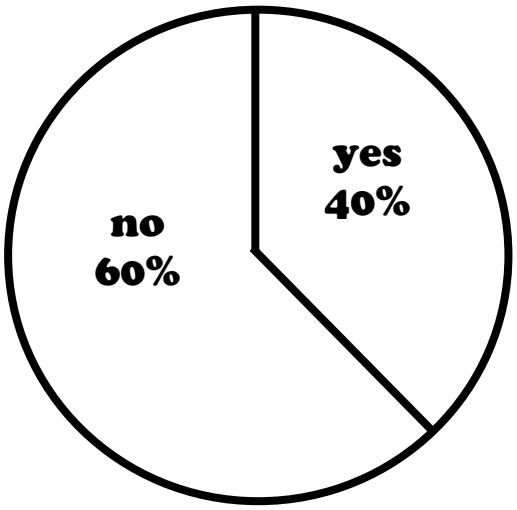
"Making Circle Graphs"

Circle graphs, or pie chars, are special kinds of graphs used to show how a whole (100%) is divided into several categories.

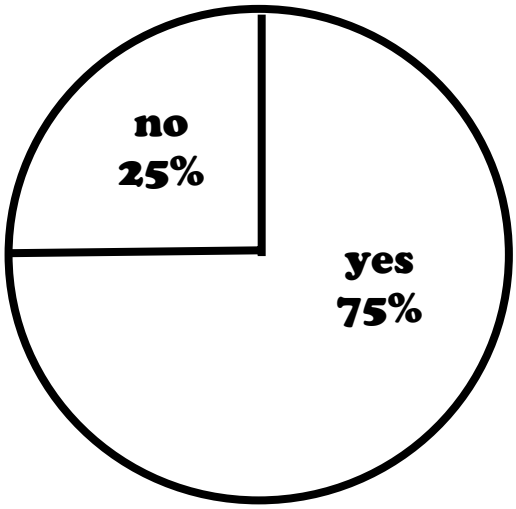
Cat and dog owners were asked, "*Do you let your pet lick your face?*" The circle graphs for this question and data are show below.

	Cat Owners	Dog Owners
yes	40%	75%
no	60%	25%
total	100%	100%

Cat Owners Who Let Their Pets Lick Their Faces



Dog Owners Who Let Their Pets Lick Their Faces



Remember that all parts of the circle (that is, all the percents) should total 100%!!!

B&PII - Problem 3.1/3.2 notes

"Getting Close & Getting Closer"

Definitions:

benchmark - **numbers used to estimate the value and location on a numberline of another point.**

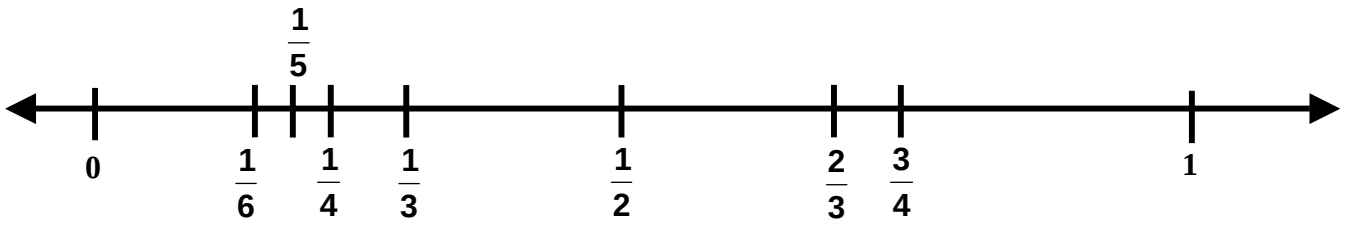
Benchmarks can be used to quickly estimate the sum of two fractions. For example, think of $\frac{1}{8} + 1\frac{5}{7}$. Is this sum larger or smaller than 2?

If you look at the number line below, you will see that $\frac{1}{8}$ is between 0 and $\frac{1}{2}$, but closer to 0 than to $\frac{1}{2}$. If you look at $1\frac{5}{7}$, you will see that it falls between $1\frac{1}{2}$ and 2 and is closer to 2. Therefore, the sum of $\frac{1}{8} + 1\frac{5}{7}$ is closest to 2.

$$0 \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad 1 \quad 1\frac{1}{4} \quad 1\frac{1}{2} \quad 1\frac{3}{4} \quad 2$$

There are several common fractions. Look at the list below and try to recall their decimal and percent equivalents. (Don't peek! 0)

Fraction	Decimal	Percent	Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%	$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$
$\frac{1}{4}$	0.25	25%	$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}\%$
$\frac{3}{4}$	0.75	75%	$\frac{1}{6}$	$0.16\overline{}$	$16\frac{2}{3}\%$
$\frac{1}{5}$	0.2	20%			

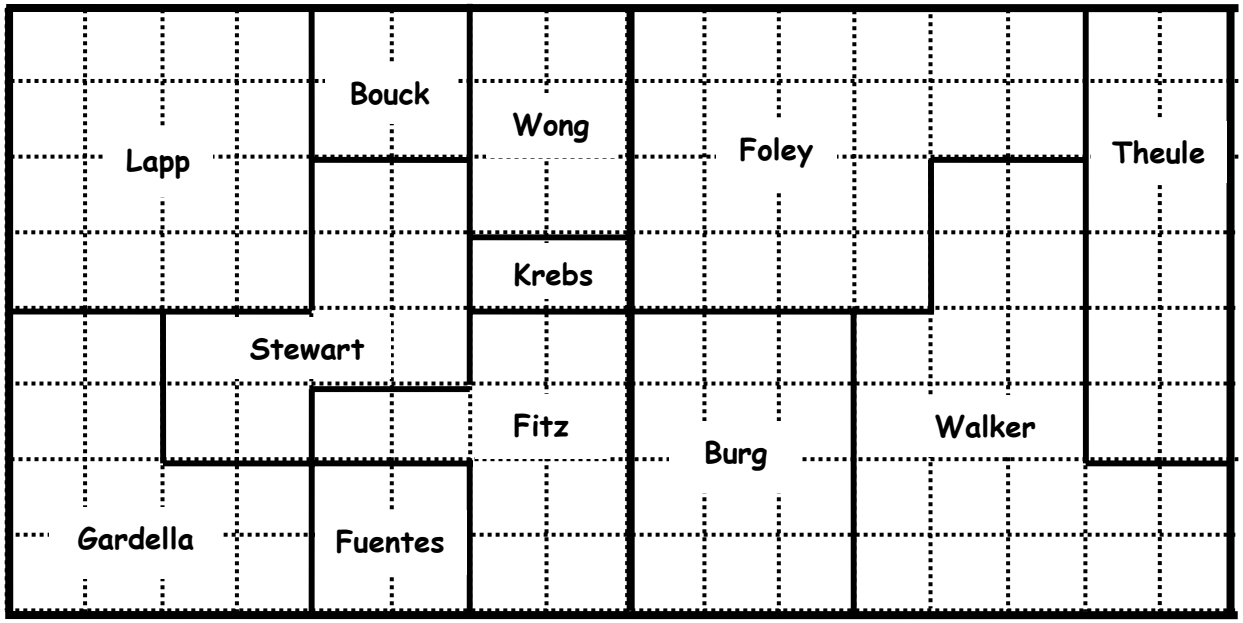


B&PII - Problem 4.1 notes

"Dividing Land"

When Tupelo Township was founded, the land was divided into sections that could be farmed. Each section is a square that is 1 mile long on each edge -- that is, each section is 1 square mile of land. There are 640 acres of land in 1-square-mile section. By dividing the sections into smaller sections, we can see what fraction of section each person owns.

Section 18 *Section 19*



Person	Land owned	Person	Land owned
Lapp	$\frac{16}{64} = \frac{1}{4}$	Bouck	$\frac{4}{64} = \frac{1}{16}$
Wong	$\frac{6}{64} = \frac{3}{32}$	Krebs	$\frac{2}{64} = \frac{1}{32}$
Stewart	$\frac{10}{64} = \frac{5}{32}$	Gardella	$\frac{12}{64} = \frac{1}{16}$
Fuentes	$\frac{4}{64} = \frac{1}{16}$	Fitz	$\frac{10}{64} = \frac{5}{32}$
Foley	$\frac{20}{64} = \frac{5}{16}$	Burg	$\frac{12}{64} = \frac{1}{16}$
Theule	$\frac{12}{64} = \frac{1}{16}$	Walker	$\frac{20}{64} = \frac{5}{16}$

B&P II Problem 4.1

Follow-Up

Determine how many acres of land each person owns.

Remember that there are 640 acres in each section, so if you divided 640 acres by 64 sections, each section is about 10 acres.

Person	Land owned	Person	Land owned
Lapp	$\frac{16}{64}$; 10 x 16 = 160	Bouck	$\frac{4}{64}$; 10 x 4 = 40
Wong	$\frac{6}{64}$; 10 x 6 = 60	Krebs	$\frac{2}{64}$; 10 x 2 = 20
Stewart	$\frac{10}{64}$; 10 x 10 = 100	Gardella	$\frac{12}{64}$; 10 x 12 = 120
Fuentes	$\frac{4}{64}$; 10 x 4 = 40	Fitz	$\frac{10}{64}$; 10 x 10 = 100
Foley	$\frac{20}{64}$; 10 x 20 = 200	Burg	$\frac{12}{64}$; 10 x 12 = 120
Theule	$\frac{12}{64}$; 10 x 12 = 120	Walker	$\frac{20}{64}$; 10 x 20 = 200

B&PII - Problem 4.2 notes

"Redrawing the Map"

As time passed, some of the owners of the land in Tupelo Township wanted to sell their land. They sold their land to others who owned land in Tupelo Township.

The following clues helped determine who owned what land.

Clue 1: When all the sales were completed, four people--Theule, Fuentes, Wong & Gardella--own all of the land in both sections.

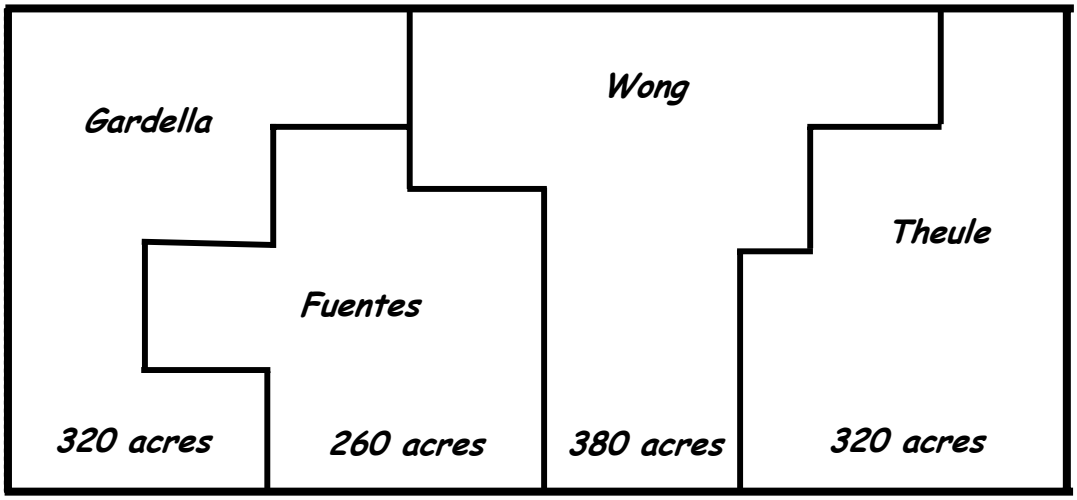
Clue 2: Theule bought from one person and now owns land equivalent to $\frac{1}{2}$ of one section.

Clue 3: Fuentes bought from three people and now owns the equivalent of $\frac{13}{32}$ of one section.

Clue 4: Gardella now owns the equivalent of $\frac{1}{2}$ section.

Clue 5: Wong now owns all of the rest of the land in the two sections.

Clue 6: Each of the four owners can walk around all of their land without having to cross onto another person's land.



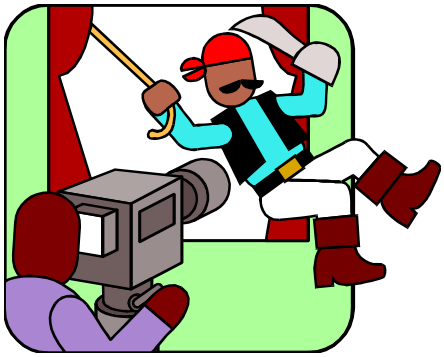
Theule bought Walker's land:	$\frac{3}{16}+\frac{5}{16}=\frac{8}{16}=\frac{1}{2}$
Fuentes bought Stewart's, Krebs', and Fitz's land:	$\frac{2}{32}+\frac{5}{32}+\frac{1}{32}+\frac{5}{52}=\frac{13}{32}$
Gardella bought Lapp's and Bouck's land:	$\frac{3}{16}+\frac{4}{16}+\frac{1}{16}=\frac{8}{16}=\frac{1}{2}$
Wong bought Foley's and Burg's land:	$\frac{3}{32}+\frac{10}{32}+\frac{6}{32}=\frac{19}{32}$

B&PII - Problem 4.3 notes

"Pirating Pizza"

Courtney's class made a gigantic square pizza for a class party to be held the day after the final exam. They made it a week before the party so they would have time to study. To keep the pizza fresh, the stored it in the cafeteria freezer.

Unfortunately, the notorious Pizza Pirate was lurking in the area. That night, the Pizza Pirate disguised himself as a janitor, tiptoed into the cafeteria and gobbled down half of the pizza. On the second night, he ate half of what was left of the pizza. Each night after that, he crept in and ate half of the pizza that remained.



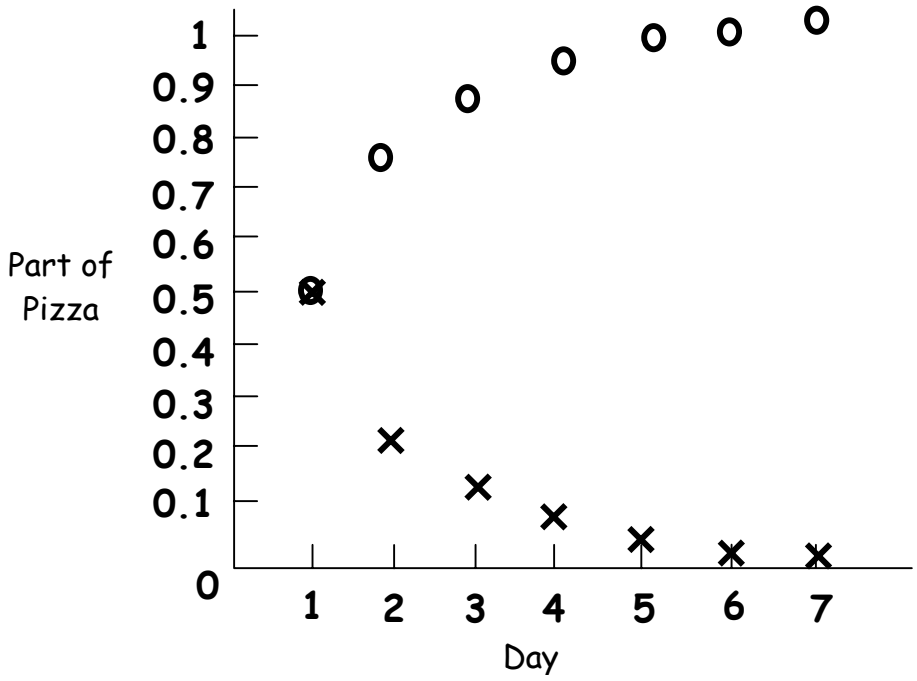
After the final exam, Courtney's class went to get their pizza to start their celebration and were stunned by what they found! How much pizza had the Pizza Pirate eaten?

Day	Amount Eaten Each Day	Total Amount Eaten	Amount Remaining
1	$\frac{1}{2}$ of $1 = \frac{1}{2}$	$\frac{1}{2}$	$1 - \frac{1}{2} = \frac{1}{2}$
2	$\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
3	$\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$	$\frac{7}{8}$	$\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$
4	$\frac{1}{2}$ of $\frac{1}{8} = \frac{1}{16}$	$\frac{15}{16}$	$\frac{1}{8} - \frac{1}{16} = \frac{1}{16}$
5	$\frac{1}{2}$ of $\frac{1}{16} = \frac{1}{32}$	$\frac{31}{32}$	$\frac{1}{16} - \frac{1}{32} = \frac{1}{32}$
6	$\frac{1}{2}$ of $\frac{1}{32} = \frac{1}{64}$	$\frac{63}{64}$	$\frac{1}{32} - \frac{1}{64} = \frac{1}{64}$
7	$\frac{1}{2}$ of $\frac{1}{64} = \frac{1}{128}$	$\frac{127}{128}$	$\frac{1}{64} - \frac{1}{128} = \frac{1}{128}$

B&P II Problem 4.3

Follow-Up

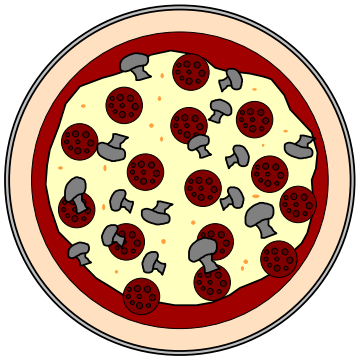
1. a) & b) The graph displays the total amount eaten so far by the Pizza Pirate for each of the seven days and how much pizza remains at the end of the day for each of the seven days.



c) *The graphs of the amount eaten and the amount remaining are reflections of each other.*

2. If the students left the pizza in freezer for a long time, would the Pizza Pirate ever eat all of the pizza?

No because the piece of pizza remaining would theoretically get smaller and smaller forever, eventually becoming microscopically small. Realistically, though, the pieces would soon become too small to matter.



B&PII - Problem 4.4 notes

"Designing Algorithms"

Definitions:

algorithm - **series of steps used for doing a computation**

In the last few Investigations, you have discovered how to add and subtract fractions. Now it is time to develop and algorithm, or formal set of steps that we can follow every time.

For adding fractions: **If the fractions have the *same* denominator, add their numerators and keep the same denominator.**

For example: $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

If the fractions have *different* denominators, find a common multiple of the two denominators, and then find an equivalent fraction for each of the fractions with a denominator that is the common multiple. Once you have equivalent fractions with like denominators, use the algorithm that helps you add two fractions with the same denominator.

For example: $\frac{4}{5} + \frac{3}{7} = \frac{28}{35} + \frac{15}{35} = \frac{43}{35} = 1\frac{8}{35}$

For subtracting fractions: **If the fractions have the *same* denominator, subtract their numerators and keep the same denominator.**

For example: $\frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

If the fractions have *different* denominators, find a common multiple of the two denominators, and then find an equivalent fraction for each of the fractions with a denominator that is the common multiple. Once you have equivalent fractions with like denominators, use the algorithm that helps you subtract two fractions with the same denominator.

For example: $\frac{4}{5} - \frac{3}{7} = \frac{28}{35} - \frac{15}{35} = \frac{13}{35}$

B&PII - Problem 5.1 notes

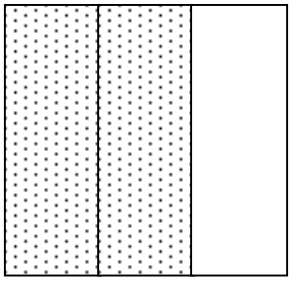
"Selling Brownies"

In the last few Investigations, you have discovered how to add and subtract fractions. Now it is time to take what you have learned and apply it to multiplication of fractions. We will start by making a model or drawing.

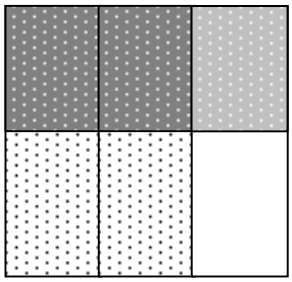
Paulo and Paula are working in the brownie booth at their school carnival. Each batch of brownies is baked in a square pan. The entire pan of brownies costs $24.

A. Mr. Sims wants to buy $\frac{1}{2}$ of a pan of brownies that is $\frac{2}{3}$ full.

The pan was $\frac{2}{3}$ full.



Mr. Sims bought $\frac{1}{2}$ of the $\frac{2}{3}$, which is $\frac{2}{6}$ of a whole pan.

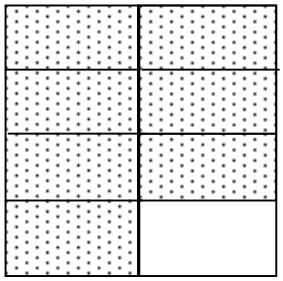


Fraction of brownies purchased is $\frac{1}{3}$ of the pan because $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$

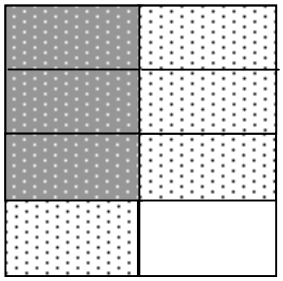
Mr. Simms spent $8 because $\frac{1}{3}$ of $24 is the same as $\frac{1}{3} \times 24 = 8$.

B. Aunt Serena wants to buy $\frac{3}{4}$ of the brownies in a pan that is half full.

The pan was $\frac{1}{2}$ full.



Aunt Serena bought $\frac{3}{4}$ of the $\frac{1}{2}$, which is $\frac{3}{8}$ of a whole pan.



Fraction of brownies purchased is $\frac{3}{8}$ because $\frac{3}{4} \times \frac{1}{2} = 3/8$

Aunt Serena spent $9 because $\frac{3}{8}$ of $24 is the same as $\frac{3}{8} \times 24 = 9$.

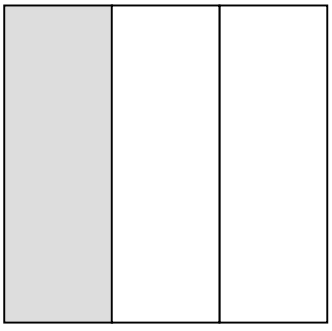
B&PII - Problem 5.2 notes

"Discounting Brownies"

When you solve problems involving multiplication with fractions, it helps to remember that finding a fraction times a number is the same as finding a fraction of a number. It is also helpful to draw models to show fraction and fraction operations.

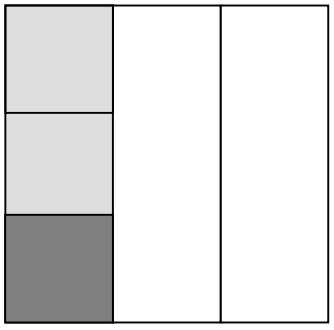
At the brownie booth, a customer wanted to buy $\frac{1}{3}$ of a pan than was $\frac{1}{3}$ full. Paula said that they had to find $\frac{1}{3}$ of $\frac{1}{3}$. Paulo said that this is the same as $\frac{1}{3}$ x $\frac{1}{3}$. They decided to make a drawing to figure out how much the customer would get.

First, they made a drawing to show how much was in the pan:



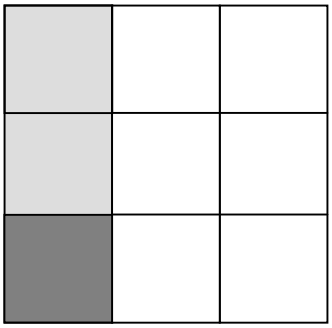
$\frac{1}{3}$ of a pan was left

Then, they showed how much the customer wanted, which was $\frac{1}{3}$ of $\frac{1}{3}$ of a pan:



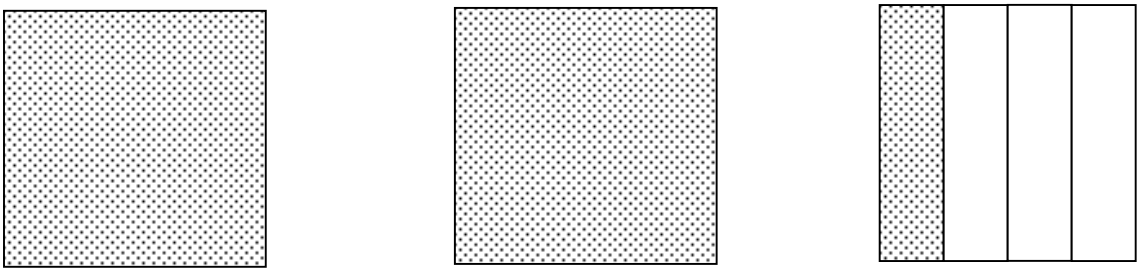
customer's part

They extended the horizontal lines to form nine equal parts. They then figured out that the customer would buy $\frac{1}{9}$ of a pan:

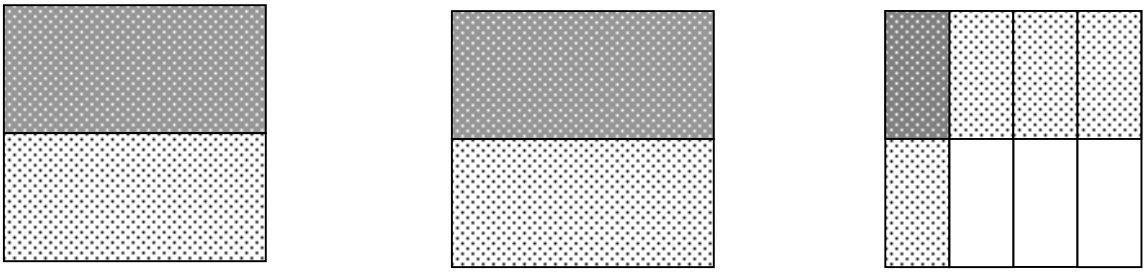


Paula & Paulo agreed that they should charge the customer $\frac{1}{9}$ of $24, or $2.67.

At the end of the carnival Paula & Paulo still had $2\frac{1}{4}$ pans of brownies remaining. They decided to offer a 20% discount. Remember that they were charging $24 per pan before the sale. Mr. Vargas wants to buy half of what is left.



There are $2\frac{1}{4}$ pans of brownies left.

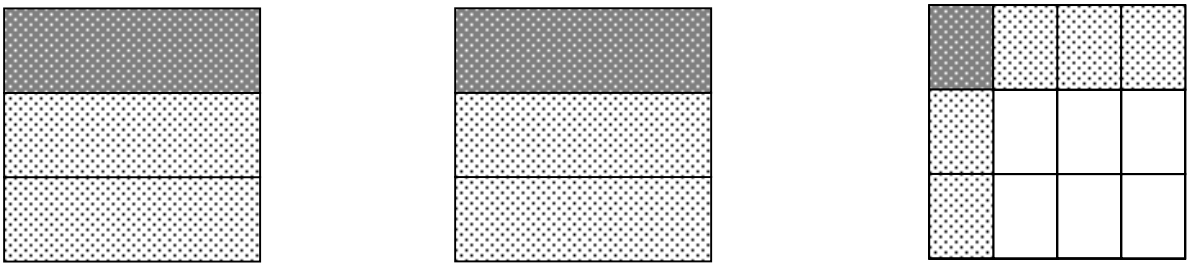


Mr. Vargas wants $\frac{1}{2}$ of the $2\frac{1}{4}$ pans, which is $\frac{1}{2}+\frac{1}{2}+\frac{1}{8}=1\frac{1}{8}$ of a pan.

Before the discount, the brownies would cost $24 + $\frac{1}{8}$ of $24 = $27.

After the discount, 20% of $27 is $5.40 and $27 - $5 .40 = $21.60.

Answer to Problem 5.2 Follow-Up



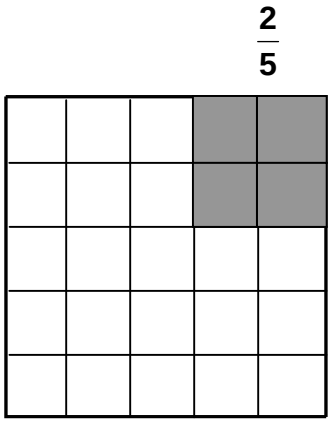
Mr. Vargas will buy $\frac{1}{3}$ of the $2\frac{1}{4}$ pans, which is $\frac{1}{3}+\frac{1}{3}+\frac{1}{12}=\frac{9}{12}=\frac{3}{4}$ of a pan.

B&PII - Problem 5.3 notes

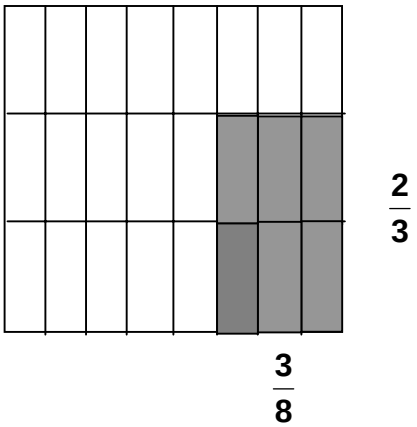
"Buying the Biggest Lot"

Miguel's mother builds and sells houses. She wants to buy a piece of land in their area on which to build several houses. There are two large lots for sale. One is a rectangular plot that is $\frac{3}{8}$ of mile by $\frac{2}{3}$ of a mile. The other is a square plot that is $\frac{2}{5}$ of a mile by $\frac{2}{5}$ of a mile.

Miguel's mother wants to know which lot is the biggest and how much she should expect to pay of the area sells for $750,000 a square mile.



The area of the plot is $\frac{4}{25}$ square mile.



The area of the plot is $\frac{6}{24} = \frac{1}{4}$ square mile.

Miguel's mother should buy the $\frac{3}{8}$ of mile by $\frac{2}{3}$ of a mile plot, because this plot, which has an area of $\frac{6}{24} = \frac{1}{4}$ of a square mile is larger than the other plot. She would expect to pay $\frac{1}{4}$ of $750,000 or $187,500.

B&PII - Problem 5.4 notes

"Designing a Multiplication Algorithm"

Recall from Investigation 4 that an algorithm is a plan, or series of steps, for doing a computation. Now we have developed an algorithm for multiplying fractions.

For multiplication of simple fractions:

If the fractions are simple (no mixed numbers) multiply the numerators together. This will be the numerator of the answer. Next, multiply the denominators together and this will be the denominator of the answer.

For example: $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$

For multiplication of mixed fractions:

If one, or both, of the fractions is a mixed number, change the whole number to a fraction with the same denominator as the fraction part. Add that fraction to the fraction part. You now have created an improper fraction. At this point, you can multiply this improper fraction and the other fraction by multiplying the numerators and the denominators.

For example: $\frac{3}{4} \times 2\frac{1}{3}$ **can be written as** $\frac{3}{4} \times \frac{7}{3}$.

$$\frac{3}{4} \times \frac{7}{3} = \frac{21}{12}$$

Be sure to simplify your final answer to $1\frac{3}{4}$.

B&PII - Problem 6.2 notes

"Moving Decimal Points"

In a decimal number, the location of the decimal point tells you the place value of every digit in the number. For example, in the numbers 236.5 and 23.65, the 2, 3, 5, and 5 mean different things. The 2 in the first number is in the *hundreds* place while the 2 in the second number is in the *tens* place.

Using only the numbers 2365 and 894, insert decimal points just before, between, or just after the given set of digits. The *order* of the digits *cannot change*. After placing the decimal point, zeros may be added only if they do not change the value of the number.

A. **Find ways to insert decimal points so you get five different sums. (some possible answers are shown)**

2353	236.5	23.65	2.365	0.2365
+ 894	+ 89.4	+ 8.94	+ 0.894	+ 0.8940
3259	325.9	32.59	3.259	1.1305

B. **Find ways to insert decimal points so you get five different differences. (some possible answers are shown)**

2353	236.5	23.65	2.365	89.400
- 894	- 89.4	- 8.94	- 0.894	- 2.365
1471	147.1	14.71	1.471	87.035

C. **Find the largest sum and the smallest sum using the same rules as before.**

Largest sum	2365	Smallest sum	0.2365
	+ 894		+ 0.8940
	3259		1.1305

D. **Find the largest difference and the smallest difference using the same rules.**

Largest difference	2365	Smallest difference	0.2365
	+ 894		+ 0.8940
	3259		1.1305

B&PII - Problem 6.3 notes

"Multiplying Decimals"

You can think of decimals as fractions with denominators of 10, 100, 1000, and so forth. For example, $\frac{1}{10}$ can be written as 0.1 and $\frac{37}{100}$ can be written as 0.37. To write the fraction $\frac{2}{5}$ as a decimal, first rewrite it as the equivalent fraction $\frac{4}{10}$, and then write it as the decimal 0.4.

Since decimal numbers are fractions, you can use what you know about multiplying fractions to help you think about how to multiply decimals. When you multiply 0.1 x 0.1 on you calculator you get 0.01. The fraction for the decimal 0.01 is $\frac{1}{100}$.

A. Each of the problems in following sets is either larger or smaller than 1 and larger or smaller than $\frac{1}{2}$.

Set 1	Set 2	Set 3	Set 4
21 x 1 is > 1	2.1 x 1 is > 1	0.21 x 1 is < ½	21 x 11 is > 1
21 x 0.1 is > 1	2.1 x 0.1 is < ½	0.21 x 0.1 is < ½	21 x 1.1 is > 1
21 x 0.01 is < ½	2.1 x 0.01 is < ½	0.21 x 0.01 is < ½	21 x 0.11 is < ½
21 x 0.001 is < ½	2.1 x 0.001 is < ½	0.21 x 0.001 is < ½	21 x 0.011 is < ½
21 x 0.0001 is < ½	2.1 x 0.0001 is < ½	0.21 x 0.0001 is < ½	21 x 0.0011 is < ½

B. The answers for each of the sets is given. Do you notice any patterns?

Set 1	Set 2	Set 3	Set 4
21 x 1= 21	2.1 x 1= 2.1	0.21 x 1= 0.21	21 x 11= 23.1
21 x 0.1= 2.1	2.1 x 0.1= 0.21	0.21 x 0.1= 0.021	21 x 1.1= 2.31
21 x 0.01= 0.21	2.1 x 0.01= 0.021	0.21 x 0.01= 0.0021	21 x 0.11= 0.231
21 x 0.001= 0.021	2.1 x 0.001= 0.0021	0.21 x 0.001= 0.00021	21 x 0.011= 0.0231
21 x 0.0001= 0.0021	2.1 x 0.0001= 0.00021	0.21 x 0.0001= 0.000021	21 x 0.0011= 0.00231

C. In a multiplication problem, there is a relationship between the number of decimal places in the factors and then number of decimal places in the product.

The number of decimal places in the answer is the sum of the number of decimal places in the factors.

B&PII - Problem 6.4 notes

"Shifting Decimal Points"

In the last investigation, you looked at how decimal points affect products. You can use what you know to work backward to find numbers with products that fit certain constraints.

A. 1. Find two numbers with a product of 1344.

possible answers: 42 x 32 = 1344 and 24 x 56 = 1344

2. Find two numbers with a product of 134.4.

possible answers: 4.2 x 32 = 134.4 and 24 x 5.6 = 134.4

3. Find two numbers with a product of 1.344.

possible answers: 0.42 x 3.2 = 1.344 and 24 x 0.056 = 1.344

4. Find two numbers with a product of 0.1344.

possible answers: 0.42 x 0.32 = 0.1344 and 0.024 x 5.6 = 0.1344

B. 1. Find two numbers with a product between 2000 and 3000.

possible answer: 56 x 48 = 2688

2. By moving decimal points, change the value of each of the numbers you found in part B1 so that their product is between 200 and 300.

possible answer: 5.6 x 48 = 268.8

3. By moving decimal points, change the value of each of the numbers you found in part B1 so that their product is between 20 and 30.

possible answer: 5.6 x 4.8 = 26.88

4. By moving decimal points, change the value of each of the numbers you found in part B1 so that their product is between 2 and 3.

possible answer: 5.6 x 0.48 = 2.688