

# Accentuate The Negative Notes

## Problem 1.1 - "Math Mania"

If a team answers a question right, they get the point value for that question. If they answer wrong, the team loses that number of points.

fractions	similarity	volume	factors	decimals	percents
50	50	50	50	50	50
100	100	100	100	100	100
150	150	150	150	150	150
200	200	200	200	200	200
250	250	250	250	250	250

At one point in the game, the scores were as follows:

Stars  
-300

Rangers  
150

**Cowboys**  
-500

- A) Which team had the highest score? *Rangers*  
lowest score? *Cowboys*
- B) How many points separate the highest and lowest score?  
*There is a 650 point difference.*
- C) Describe a sequence of 3 events that could have given the Stars team their score. *They could have answered a 50 point question wrong, then a 100 point question wrong, then a 150 point question wrong*
- D) The teams keep playing, and the following happens:  
Stars: 200 pt question correct, 150 pt question incorrect,  
50 pt question correct, and a 50 pt question correct.

What is the team's new score? *They started at -300, then adding 200 they now have -100, then losing 150 they have -250, then adding 50 they have -200, then adding 50 more, they end up at -150.*

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Problem 1.2 - "Math Mania - Winning the Game"

Another group of students played the Math Mania game. There were five teams, and their final scores were:

Team A: 200	Team B: -250	Team C: -400	Team D: 350	Team E: -100
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A) Order the teams by score, from 1st place to 5th place.

*team D(350), team A(200), team E(-100), team B(-250), and team C(-400)*

B) How many points is 1st place ahead of 2nd place?

*200 → 350 = 150 points ahead*

C) How many points is 1st place ahead of 3rd place?

*-100 → 350 = 450 points ahead*

D) How many points is 2nd place ahead of 4th place?

*-250 → 200 = 450 points ahead*

E) How many points is 3rd place ahead of 5th place?

*-400 → -100 = 300 points ahead*

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## Problem 1.3 - "Measuring Temperature"

Definitions:

**opposites** - numbers that are the same distance from zero on the number line.  
(EX: -3 and 3, -5 and 5, ...)

A thermometer can be thought of as a vertical number line. The positive numbers are the temperatures above 0 and the negative numbers are the temperatures below 0.

A. Arrange the temperatures  $-8^\circ$ ,  $4^\circ$ ,  $12^\circ$ ,  $-2^\circ$ ,  $0^\circ$ ,  $-15^\circ$  from lowest to highest.      *-15, -8, -2, 0, 4, 12*

B. If the temperature begins at  $5^\circ\text{F}$ , what will be the new reading if the temperature...

rises  $10^\circ$ :       $5 + 10 = 15^\circ$

falls  $10^\circ$ :  $5 - 10 = -5^\circ$

C. If the temperature begins at  $-5^\circ$ , what will be the new reading if the temperature...

falls  $10^\circ$ :       $-5 - 10 = -15^\circ$

rises  $10^\circ$ :  $-5 + 10 = 5^\circ$

D. Give the temperature halfway between the given temps:

1.  $0^\circ$  and  $10^\circ$ :  $5^\circ$

2.  $-5^\circ$  and  $15^\circ$ :  $5^\circ$

3.  $5^\circ$  and  $-15^\circ$ :  $-5^\circ$

4.  $0^\circ$  and  $-20^\circ$ :  $-10^\circ$

5.  $-8^\circ$  and  $8^\circ$ :  $0^\circ$

6.  $-6^\circ$  and  $-16^\circ$ :  $-11^\circ$

E. Tell which temperature is farther from  $-2^\circ$ .

1.  $-6^\circ$  or  $6^\circ$ ?       $6^\circ$

2.  $-7^\circ$  or  $3^\circ$ ?      *same*

3.  $2^\circ$  or  $-5^\circ$ ?       $2^\circ$

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## Problem 2.1 - "Adding on a Number Line"

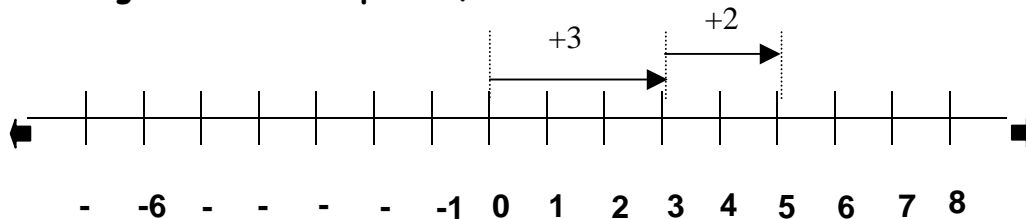
definitions:

**integers**- the set of whole numbers and their opposites (...-3,-2,-1,0,1,2,3...)

**number sentence**- a numerical representation of a mathematical situation

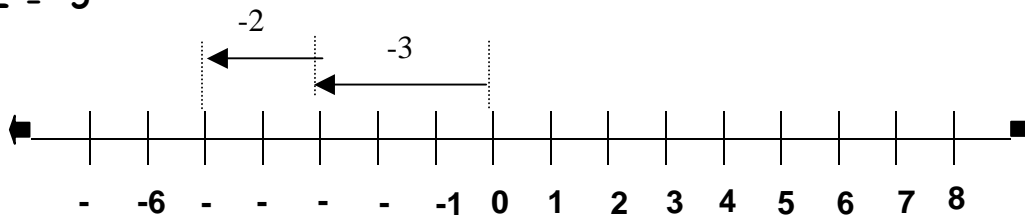
Monique came up with following method for represent the number sentence  $3 + 2 = 5$ :

**Start at 0, and move 3 units to the right. To show the addition of 2, move 2 more units to the right. You end up at 5, so  $3 + 2 = 5$**



Ethan thought a similar method would work for adding negative integers. He came up with the following plan for finding  $-3 + -2$ .

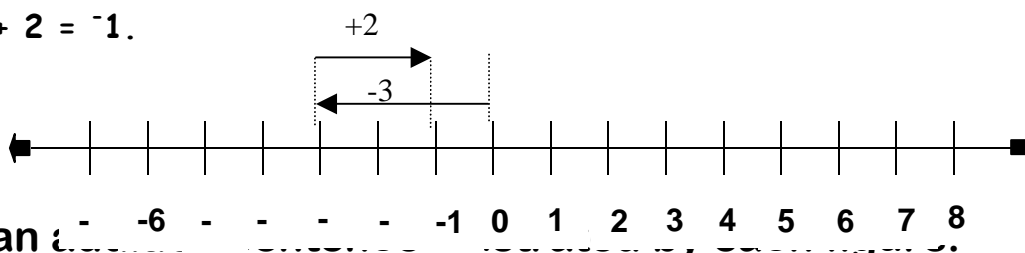
**Start at 0, and move 3 units to the left (the negative direction) to represent the -3. To show the addition of -2, move 2 more units to the left. You end up at -5, so  $-3 + -2 = -5$**



When Ethan wrote  $-3 + -2 = -5$ , he used raised negative symbols to help him separate the sign of the integer from the operation sign for addition.

Then, Ethan wanted to try adding a negative integer and a positive integer on a number line. He followed these steps to find  $-3 + 2$ :

**Start at 0, and move 3 units to the left (the negative direction) to represent -3. To show the addition of 2, move two units to the right (the positive direction). You end up at -1, so  $-3 + 2 = -1$ .**



A. Write an addition sentence for each number line.

**illustration**

**addition sentence**

	$-3 + -4 = -7$
	$-3 + 4 = 1$
	$-4 + 11 = 7$
	$5 + -8 = -3$

**B. When you add two integers, does the order of the numbers make a difference? Use the following examples to test your conclusion.**

1.  $-5 + 10$  and  $10 + -5$

2.  $-4 + -6$  and  $-6 + -4$

3.  $8 + -8$  and  $-8 + 8$

4.  $6 + -7$  and  $-7 + 6$

**When adding integers, the order does not matter.**

The answers to the above problems are:

1. 5

2. -10

3. 0

4. -1

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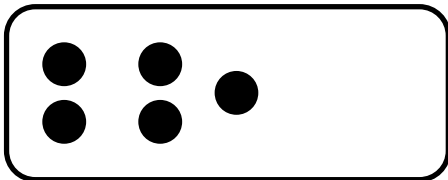
## Problem 2.2 - "Adding on a Chip Board"

- ⇒ red chips (or white chips) represent negative numbers (one chip = -1)
- ⇒ black chips represent positive numbers (one chip = +1)
- ⇒ a red chip and a black chip together make a zero pair ( $-1 + 1 = 0$ )
- ⇒ after all chips are added to the chip board, you can remove sets of zero pairs to get the answer to the addition problem

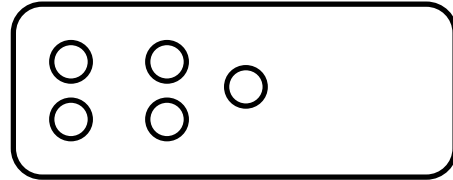
Positive one (1) is ●

negative one (-1) is ○

This chip board shows a value of 5:

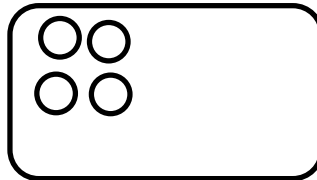


This chip board shows a value of -5:

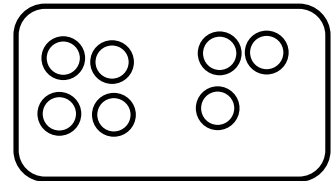


To represent  $-4 + -3$ , start with an empty chip board.

*Represent -4 by putting four white chips on the board.*



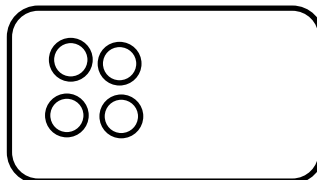
*Represent the addition of -3 by putting three more white chips on the board.*



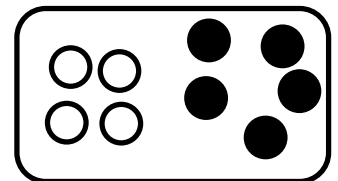
Since there were seven white chips on the board, you can conclude that the sum of  $-4$  and  $-3$  is  $-7$ . Written in a number sentence it would be  $-4 + -3 = -7$ .

To represent  $-4 + 5$ , start with a clear board.

*Represent -4 by putting four white chips on the board.*

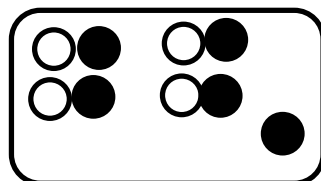


*Represent the addition of 5 by putting five black chips on the board.*



Since 1 and -1 are opposites, they have a value of 0. So a black chip and a white chip have a value of zero (zero pairs).

*Find all sets of zero pairs.*



*Your answer is the total number of chips not in a zero pair.*



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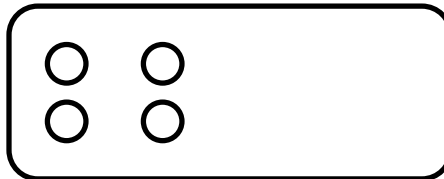
## Problem 3.1 - "Subtracting on a Chip Board"

- ⇒ red chips (or clear chips) represent negative numbers (one chip = -1)
- ⇒ black chips represent positive numbers (one chip = +1)
- ⇒ a red chip and a black chip together make a zero pair ( $-1 + 1 = 0$ )

### ***STEPS FOR SUBTRACTING ON A CHIP BOARD:*** (using the example $-4 - 5$ )

1. put on chips to represent the first number of the problem

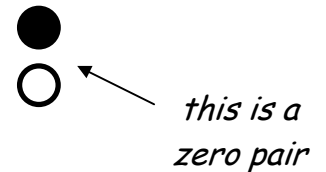
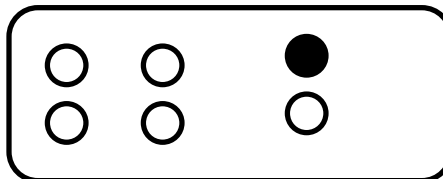
*put on 4 negatives*



2. take off the chips that represent the second number

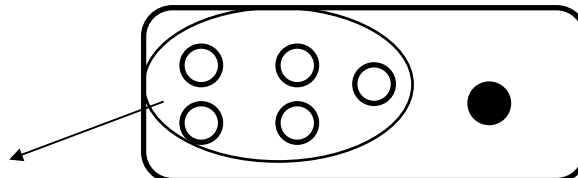
⇒ if the second number of chips cannot be removed, add zero pairs

*we only had 4 negatives  
so we need one more -  
add one zero pair*



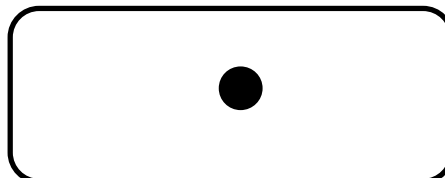
3. when enough zero pairs have been added, go back to step 2

*now we can take of  
our 5 negatives*



4. the answer is the number of chips remaining

*answer = +1*



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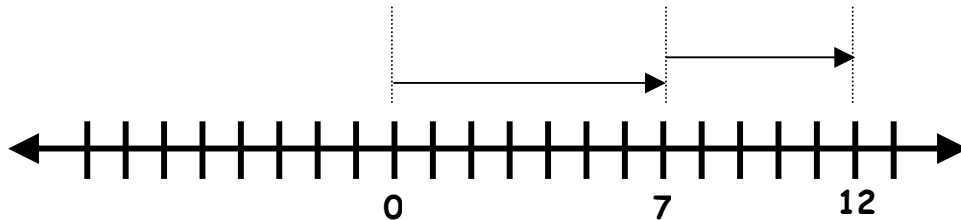
## Problem 3.2 - "Subtracting on a Number Line"

Adding and subtracting can be thought of as *inverse operations* (inverse means opposite). Subtraction can be used to undo addition, and addition can be used to undo subtraction. This means that what we normally do when adding, we will do the opposite of when subtracting.

Recall the method for showing an addition problem on a number line:

$$7 + 5 = ?$$

Begin at zero, move 7 spaces to the right, then move 5 more spaces to the right.

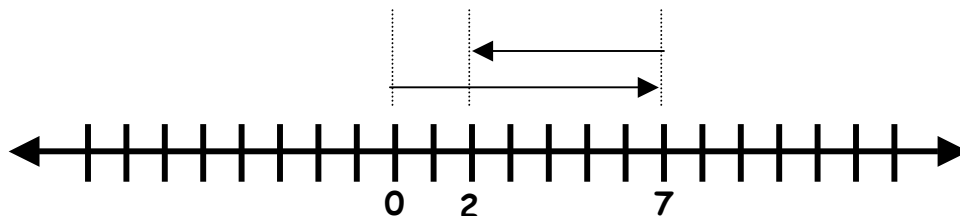


The solution is 12.

To model a subtraction problem, use the idea of inverse operations.

$$7 - 5 = ?$$

Begin at zero, move 7 spaces to the right, then move 5 spaces to the **left** (because we are doing the **OPPOSITE** of addition, we move in the **OPPOSITE** direction).

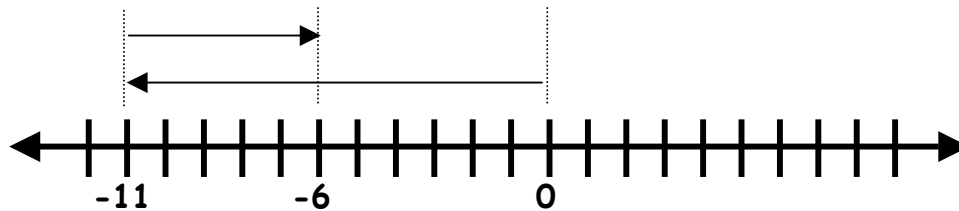


The solution is 2.

Now let's try another problem with negatives.

$$^{-}11 - ^{-}5 = ?$$

Begin at zero and move 11 space to the *left*. Now, if we were ADDING -5 we would move to the left, but since we are SUBTRACTING -5, we are going to do the OPPOSITE. This means to subtract -5, we must move 5 spaces to the *right*.

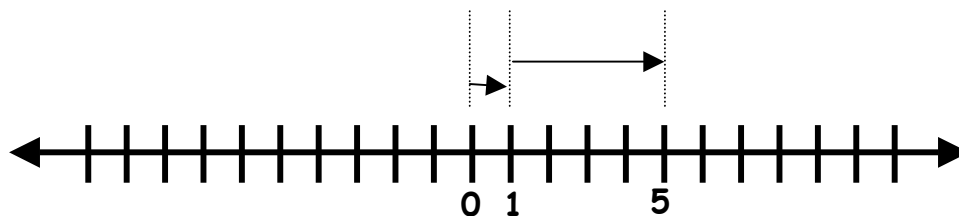


The solution is -6.

Let's look at one more example of subtracting a negative.

$$1 - ^{-}4 = ?$$

Begin at zero and move 1 space to the *right*. Now, if we were ADDING -4 we would move to the left, but since we are SUBTRACTING -4, we are going to do the OPPOSITE. This means to subtract -4, we must move 4 spaces to the *right*.



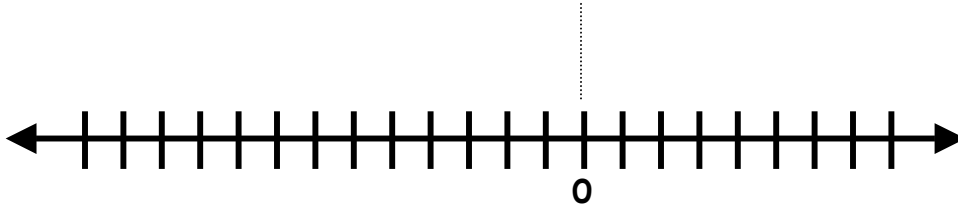
The solution is 5.

Now try one on your own.

$$^{-}8 - 4 = ?$$

Begin at zero and move 8 space to the *left*. Now, if we were ADDING 4 we would move to the right, but since we are SUBTRACTING 4, we are going to do the OPPOSITE. This means to subtract 4, we must move 4 spaces to the *left*.

Show this on a number line:



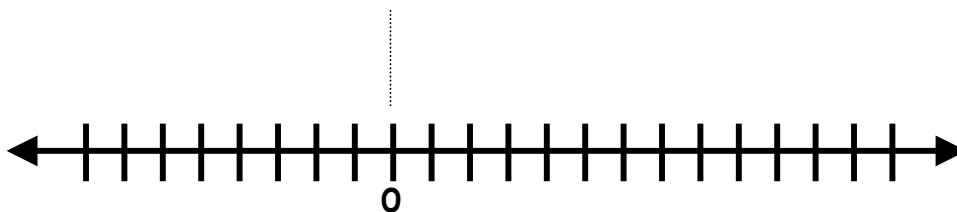
The solution is \_\_\_\_\_.

Now try another problem with negatives.

$$3 - ^{-}9 = ?$$

Begin at zero and move 3 spaces to the *right*. Now, if we were ADDING -9 we would move to the left, but since we are SUBTRACTING -9, we are going to do the OPPOSITE. This means to subtract -9, we must move 9 spaces to the *right*.

Show this on a number line:



The solution is \_\_\_\_\_.

# Accentuate The Negative Notes

## Problem 3.3 - "Exploring Patterns"

- ⇒ Study the pattern in the subtraction problems below.  
⇒ Think about WHAT is changing and HOW it is changing.

$$15 - 5 = 10$$

$$15 - 4 = 11$$

$$15 - 3 = 12$$

$$15 - 2 = 13$$

$$15 - 1 = 14$$

$$15 - 0 = 15$$

each time the number  
you are subtracting  
decreases by one, the  
answer increases by one.



Use the pattern you found to predict the answers to the following problems:

$$15 - 1 =$$

$$15 - 2 =$$

$$15 - 3 =$$

⇒ Now, look at the next set of equations and study the pattern.

⇒ Finish by predicting the answers to the last 2 problems.

$$-10 - 3 = -13$$

$$-10 - 2 = -12$$

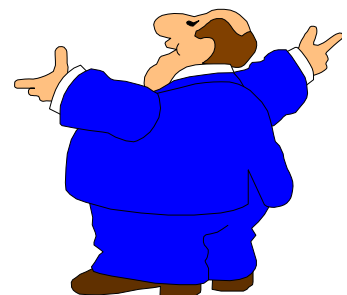
$$-10 - 1 = -11$$

$$-10 - 0 = -10$$

$$-10 - 1 =$$

$$-10 - 2 =$$

it's the same pattern!  
each time the number  
you are subtracting  
decreases by one, the  
answer increases by one



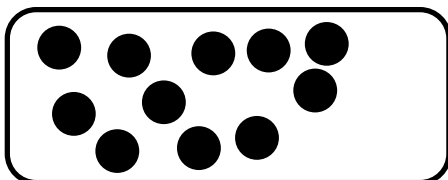
# Accentuate The Negative Notes

## Problem 3.4 - "Undoing Addition & Subtraction"

### ***STEPS FOR "UNDOING" ON A CHIP BOARD:***

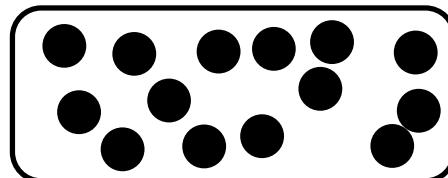
(using the example  $11 + 3 = 14$ )

1. Place eleven chips on the board represent the first number of the problem.



*put on 11 positives*

2. Add three more black chips to the board.



*add 3 more positives*

If you removed 3 black chips, you would end up with 11 chips. In other word, removing chips from the board "undoes" placing chips on the board. Original problem:  $11 + 3 = 14$

Subtractions "undoes" addition, so the following problem with "undo" the original problem:  $11 = 14 - 3$

- A. The addition sentence is:  $-17 + 13 = -4$   
The subtraction sentence is:  $-17 = -4 - 13$

- B. The addition sentence is:  $-4 + -18 = -22$   
The subtraction sentence is:  $-4 = -22 - -18$

- C. 1. addition:  $24 + -18 = 6$       2. addition:  $-28 + -13 = -41$   
subtraction:  $6 - -18 = 24$       subtraction:  $-41 - -13 = -28$

3. addition:  $-9.3 + 6.1 = -3.2$       4. addition:  $\frac{2}{3} + \frac{-1}{3} = \frac{1}{3}$   
subtraction:  $-3.2 - 6.1 = -9.3$       subtraction:  $\frac{1}{3} - \frac{-1}{3} = \frac{2}{3}$

- D. 1. subtraction:  $-12 - -6 = -6$       2. subtraction:  $1 - -2 = 3$   
addition:  $-6 + -6 = -12$       addition:  $3 + -2 = 1$

3. subtraction:  $-1.8 - 5.3 = -7.1$       4. subtraction:  $-1 - \frac{-3}{4} = \frac{-1}{4}$   
addition:  $-7.1 + 5.3 = -1.8$       addition:  $\frac{-3}{4} + \frac{-1}{4} = -1$

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## Problem 4.1 - "Rising & Falling Temperatures"

In this investigation, you will use a positive symbol to represent a rise in temperature and a negative symbol to represent a drop in temperature.

- A. Suppose the temperature changed by an average of  $3^\circ$  per hour for a 40-hour period. Complete the table below and use it to find the total temperature change for the first 5 hours.

<b># of hours</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>Total temp. change</b>	<b><math>+3^\circ</math></b>	<b><math>+6^\circ</math></b>	<b><math>+9^\circ</math></b>	<b><math>+12^\circ</math></b>	<b><math>+15^\circ</math></b>

Write a multiplication sentence that represents the total change in temperature for the first 5 hours.  **$5 \times +3^\circ = +15^\circ$**

Write a multiplication sentence that represents the total change in temperature for all 10 hours.  **$10 \times +3^\circ = +30^\circ$**

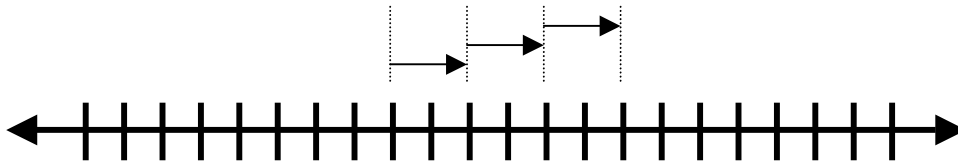
- B. Suppose the temperature changed by an average of  $-3^\circ$  per hour for a 10-hour period. Complete the table below and use it to find the total temperature change for the first 5 hours.

<b># of hours</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>Total temp. change</b>	<b><math>-3^\circ</math></b>	<b><math>-6^\circ</math></b>	<b><math>-9^\circ</math></b>	<b><math>-12^\circ</math></b>	<b><math>-15^\circ</math></b>

Write a multiplication sentence that represents the total change in temperature for the first 5 hours.  **$5 \times -3^\circ = -15^\circ$**

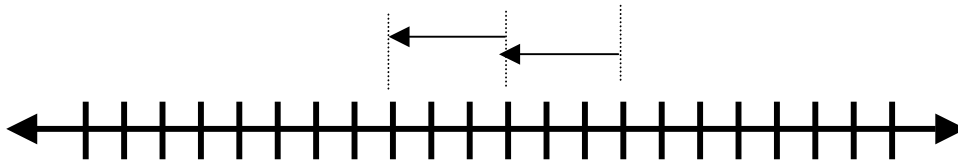
Write a multiplication sentence that represents the total change in temperature for all 10 hours.  **$10 \times -3^\circ = -30^\circ$**

C. Write an addition sentence and a multiplication sentence for each illustration below.



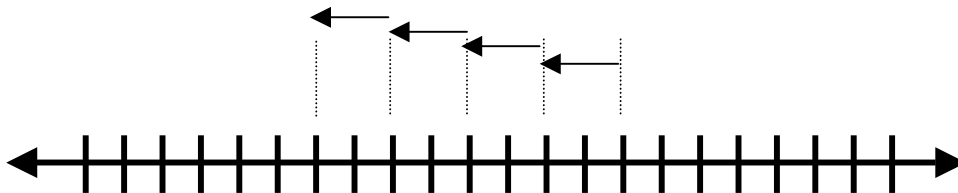
Addition sentence:  $2 + 2 + 2 = 6$

Multiplication sentence:  $3 \times 2 = 6$



Addition sentence:  $-3 + -3 = -6$

Multiplication sentence:  $2 \times -3 = -6$



Addition sentence:  $-2 + -2 + -2 + -2 = -8$

Multiplication sentence:  $4 \times -2 = -8$

D. Make up a situation about temperature that can be expressed as  $4 \times -10$ .

*The temperature drops 10 degrees for each of the next 4 hours.*

E. Find each product.

1.  $5 \times -4 = -20$

2.  $20 \times -4 = -80$

3.  $-4 \times 20 = -80$

4.  $-5 \times 4 = -20$

# Accentuate The Negative Notes

## Problem 4.2 - "Exploring Patterns"

- ⇒ Study the pattern in the subtraction problems below.  
⇒ Think about WHAT is changing and HOW it is changing.

$$5 \times 5 = 25$$

$$5 \times 4 = 20$$

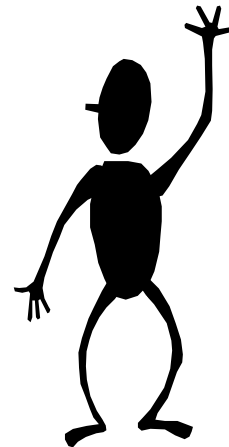
$$5 \times 3 = 15$$

$$5 \times 2 = 10$$

$$5 \times 1 = 5$$

$$5 \times 0 = 0$$

each time the number  
you are multiplying  
decreases by one, the  
answer decreases by five.



Use the pattern you found to predict the answers to the following problems:

$$5 \times -1 =$$

$$5 \times -2 =$$

$$5 \times -3 =$$

⇒ Now, look at the next set of equations and study the pattern.

⇒ Finish by predicting the answers to the last 2 problems.

$$-4 \times 3 = -12$$

$$-4 \times 2 = -8$$

$$-4 \times 1 = -4$$

$$-4 \times 0 = 0$$

$$-4 \times -1 =$$

$$-4 \times -2 =$$

it's the same pattern!  
each time the number  
you are multiplying  
decreases by one, the  
answer decreases by four



# Accentuate the Negative Notes

## Problem 4.4 - "Dividing Integers"

Earlier you learned that subtraction is the **INVERSE** of addition. Division is the inverse of multiplication. For example, you can write two division sentences that will undo the multiplication problem  $5 \times 6 = 30$ .

$$30 \div 6 = 5 \quad \text{or} \quad 30 \div 5 = 6$$

1. A. Complete the multiplication sentence -  $5 \times 6 = -30$   
B. Write two division sentences that are equivalent to the multiplication sentence you find in part A.

$$-5 = -30 \div 6 \quad 6 = -30 \div -5$$

2. A. Complete the multiplication sentence -  $8 \times -4 = 32$   
B. Write two division sentences that are equivalent to the multiplication sentence you find in part A.

$$-4 = 32 \div -8 \quad -8 = 32 \div -4$$

3. Write a division sentence that solves each problem.

a)  $? \times 12 = -132$   
 $-132 \div 12 = -11$

b)  $-8 \times ? = -56$   
 $-56 \div -8 = 7$

c)  $? \times -4 = 132$   
 $132 \div -4 = -33$

d)  $5.2 \times ? = -8.84$   
 $-8.84 \div 5.2 = -1.7$

4. Write a division or multiplication sentence that solves each problem.

a)  $? \div -3 = -8$   
 $-8 \times -3 = 24$

b)  $91 \div ? = -7$   
 $91 \div -7 = -13$

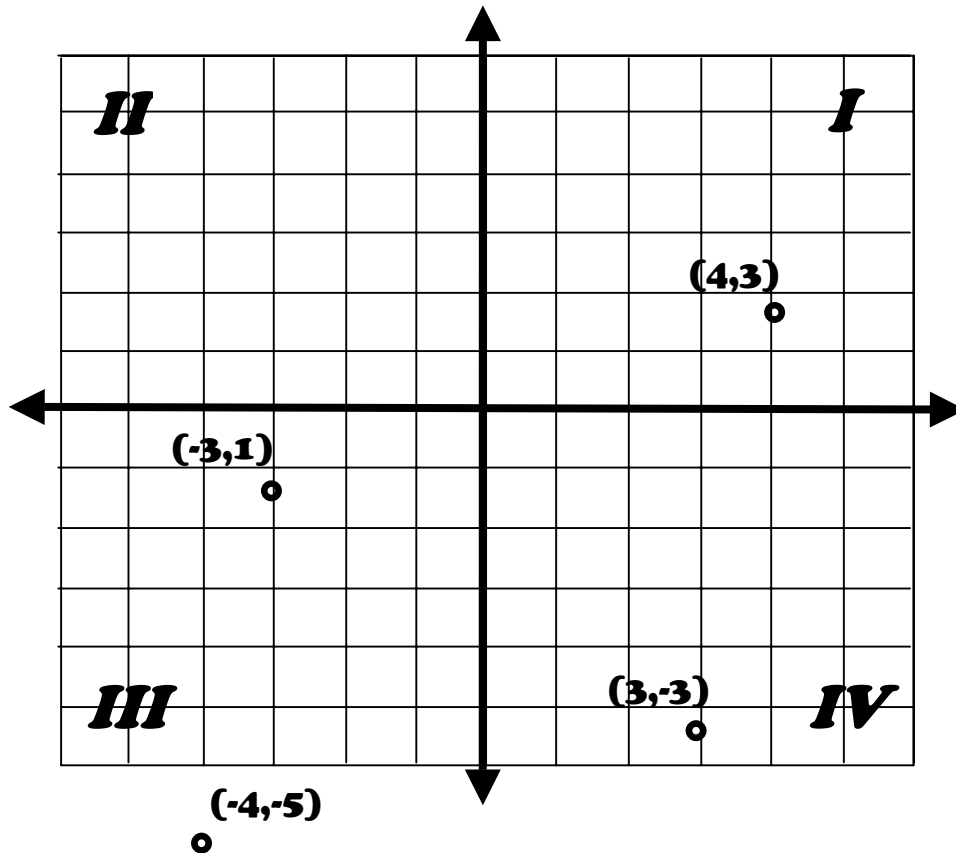
c)  $? \div 11 = -17$   
 $-17 \times 11 = -187$

d)  $-19.95 \div ? = 9.5$   
 $-19.95 \div 9.5 = -2.1$

# Accentuate The Negative Notes

## Problem 5.1 - "Coordinate Grids"

The x-axis on a coordinate grid is a horizontal number line, and the y-axis is a vertical number line.



- The 4 sections of the coordinate grid are called quadrants (I, II, III, and IV).
- Each (x,y) ordered pair is called a coordinate.

*The following coordinates are located and labeled on the graph:*

*(4, 3), (-3, 1), (-4, -5), and (3, -3)*

- Quadrant I is at the upper right, then they are numbered counterclockwise.  
*We start counting at this section because both the x and y coordinates are positive numbers.*
- The center point (0,0) is called the origin.  
*We call this the origin because this is the point where both values are zero.*