

Shapes & Designs Notes

Problem 1.1

Definitions:

regular polygons - polygons in which all the side lengths and angles have the same measure

edge - also referred to as the side of a figure

tiling - covering a flat surface with shapes that fit together without any gaps or overlaps

Each of the figures below are **regular polygons**. This means that each of the angles and side lengths in a figure are all the same size. Some, not all, of the figures will **tile**. This means that you can use the same shape over and over again to create a pattern without any gaps or overlaps. It is also possible to combine shapes to tile a pattern.

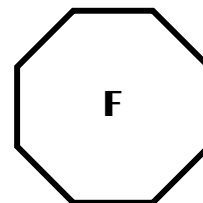
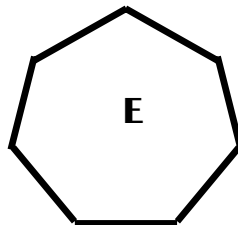
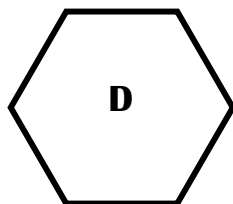
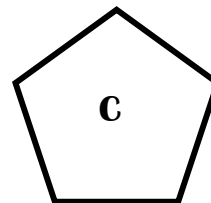
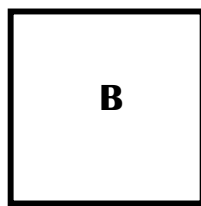
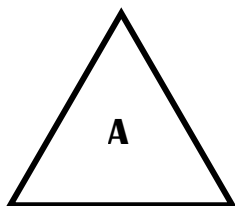


figure	shape name	# of sides
A	triangle	3
B	square	4
C	pentagon	5
D	hexagon	6
E	heptagon	7
F	octagon	8

- A. Which polygons will create a tiling using only one type of polygon?
Squares, triangles, and hexagons can be used alone to cover a surface. The pentagon, heptagon, and octagon will not tile a surface.
- B. Which polygons will create a tiling using two or more different polygons?
Some combinations that will tile are octagons and squares, hexagons and triangles, and squares and triangles.

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Problem 2.1

Triangles are the simplest polygons. Although they have only three sides and three angles, triangles come in many different shapes with very useful properties. You can use polystrips to make triangles with different side lengths.

Think about the following questions:

- **If you are given three side lengths, can you always make a triangle?**
- **With three side lengths, can you make more than one triangle?**

Explore these questions by first selecting three numbers between 1 and 20 and using polystrips to make a triangle with the numbers as side lengths.

- A. Will it always be possible to make a triangle with those side lengths?
No; not every combination of three sides will make a triangle. To make a triangle, the sum of the lengths of any two sides must be greater than the length of the third side.
- B. Can you make two or more different triangles from the same side lengths?
There is only one triangle possible from three side lengths. (This is what gives triangles the stability that makes them useful in buildings and other structures.)

Follow-Up

1. What combinations of side lengths give triangles like those you see often in designs and buildings?
Some possible answers are: three sides that are the same length; two sides that are the same length.
2. What combinations of side lengths give triangles with strange shapes?
Some possible answers: one long edge and two short edges; three sides that are very different lengths.
3. What combinations of side lengths give triangles that have symmetry?
Some possible answers: triangles that look more regular; triangles with all three sides equal; triangles with two sides equal.

Shapes & Designs Notes

Problem 2.2

Definitions:

quadrilateral - a polygon with four sides

Quadrilaterals are four-sided polygons. Many different kinds of quadrilaterals appear in the structures and designs all around us. You can use polystrips to make quadrilaterals with different side lengths. Make a note of the ways quadrilaterals and triangles are similar and ways they are different.

Think about the following question:

- **How are quadrilaterals and triangles similar? How are they different?**

Explore these questions by first selecting three numbers between 1 and 20 and using polystrips to make a quadrilateral with the numbers as side lengths.

- A. Will it always be possible to make a quadrilateral with those side lengths?
No; no side can be equal to or greater in length than the sum of the lengths of the other three sides.
- B. Can you make two or more different quadrilaterals from the same side lengths?
Yes; many different quadrilaterals can be made with the same set of sides by changing the order of the sides or positioning the angles differently.

Follow-Up

1. What combinations of side lengths give quadrilaterals like those you see often in designs and buildings?
Squares and rectangles are quadrilaterals. They both have all 90 degree angles, but squares have side lengths all the same measure. Diamonds and parallelograms are also quadrilaterals, but they do not have 90 degree angles.
2. What combinations of side lengths give quadrilaterals with strange shapes?
One possible answer is a quadrilateral with no sides the same length.
3. What combinations of side lengths give quadrilaterals that have symmetry?
Some possible answers: squares, rectangles, and isosceles trapezoids.

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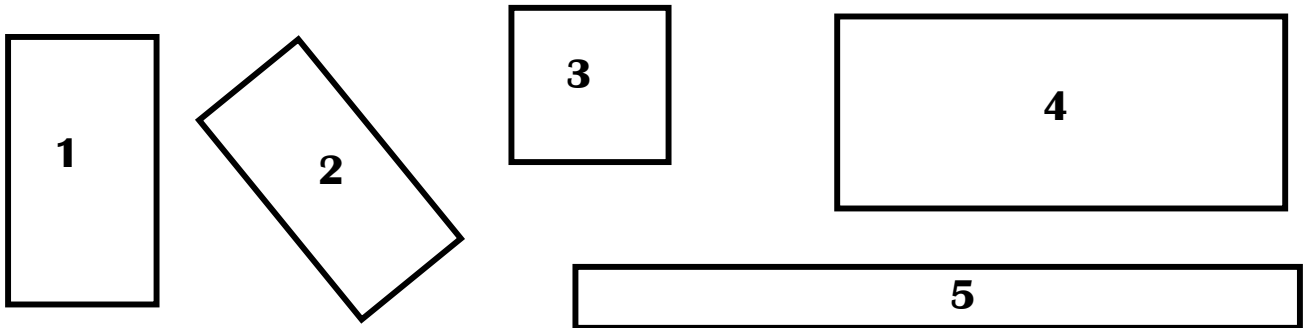
Problem 2.3

Definitions:

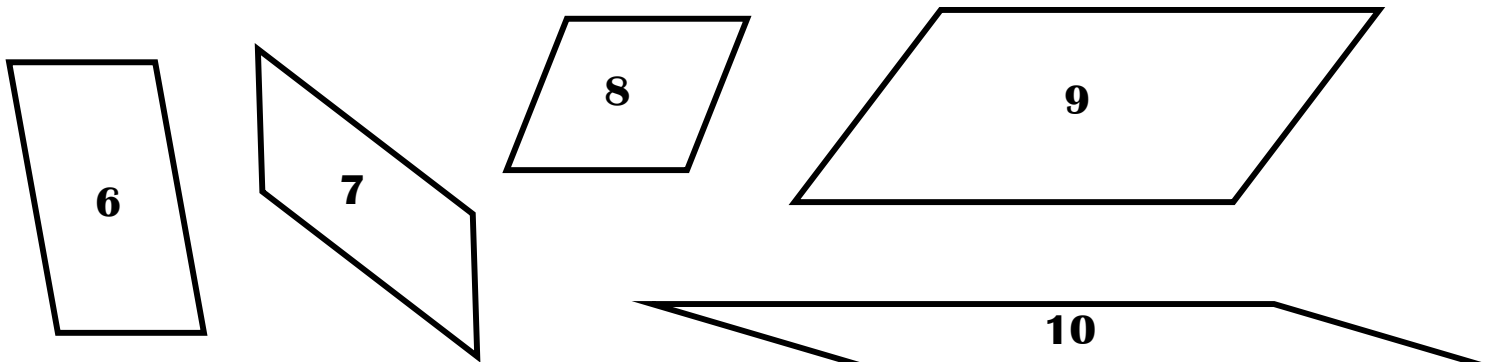
parallelograms - a quadrilateral with two sets of parallel lines

parallel lines - straight lines that never meet, no matter how far they are extended

Rectangles may be the most common quadrilaterals. You can find them in buildings and designs everywhere. Here are five examples of rectangles.



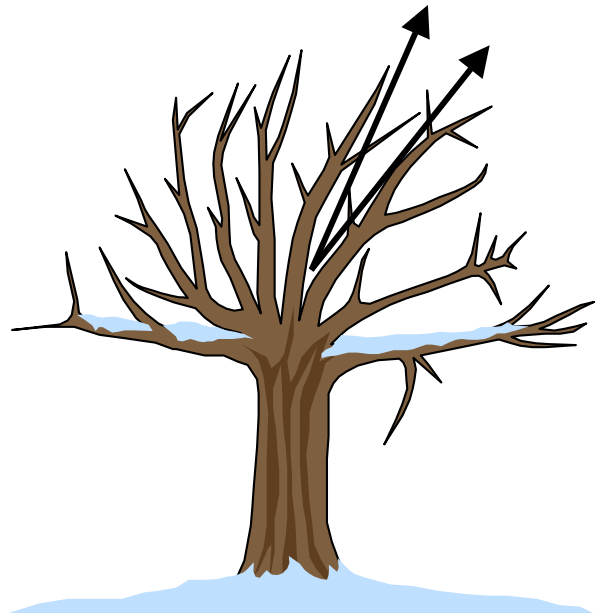
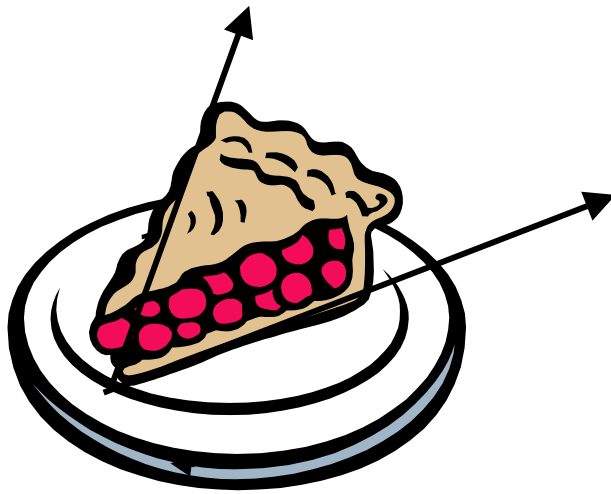
You probably have discovered that if you build a rectangle and push on one of its corners, it easily changes into different shapes such as the ones below. These are called **parallelograms**.



- A. What do these ten quadrilaterals have in common that makes the name parallelogram sensible? *Their opposite sides are parallel.*
- B. How do rectangles 1-5 differ from shapes 6-10 (which were formed by pressing on the corners of 1-5)? *Rectangles 1-5 all have only right angles.*
- C. How are the lengths of the sides of a parallelogram related? *Both pairs of opposite sides are equal in length.*

The bee dance illustrates one way that you can think about an angle - as a **turn**. When the honey bee dances along a tilted line, she is telling the other bees how far to turn from the sun to find the flowers.

You can also think about an angle as a **wedge**, like a piece of pizza. Finally you can think about an angle as two sides that meet at one point, like branches on a tree. This point is called the **vertex**.



A. An angle that occurs as the result of a turning motion?

Some possible answers: a car or bike changing direction; members of a band making turns; swinging of baseball bat or golf club.

B. An angle that occurs as a wedge, such as a piece of pizza?

Some possible answers: pizza or pie sections; light from a flashlight; tip of a pencil.

C. An angle that occurs as two sides with a common vertex, such as the branches on a tree?

Some possible answers: the edges of a floor, ceiling, or window; the hands of a clock; the blades of an open pair of scissors; two fingers spread apart.

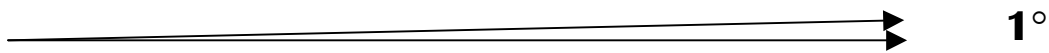
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Problem 3.2

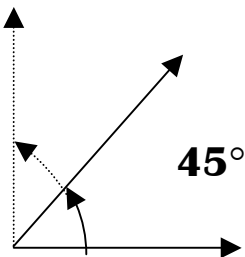
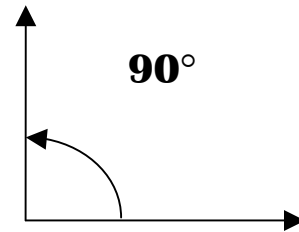
Definitions:

degree - a unit of measure used to describe an angle's measure

There are several ways to describe the size of an angle. The most common way is the **degree**. An angle of 1 degree (also written as 1°) is a very small turn, or a very narrow wedge.



The size of the degree was chosen so that a right angle has a measure of 90° . Here is a 90° angle. Imagine 90 copies of the 1° angle fitting into this angle.

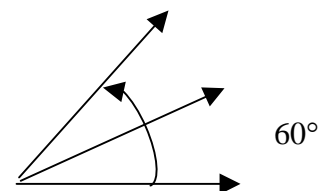
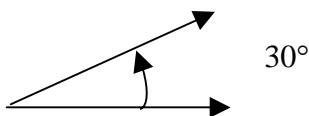


Here is an angle formed by one half the turn of a right angle. It measures 45° .

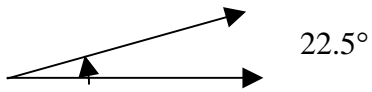
It is often useful to estimate the measures of angles and to sketch angles when a measurement is given. The degree measures of the angles made by the described turns are shown below.

A. One-third of a right-angle turn.

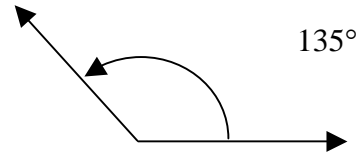
B. Two-thirds of a right-angle turn.



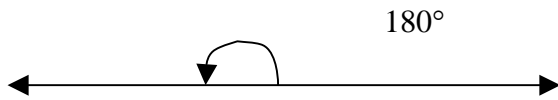
C. One-quarter of a right-angle turn.



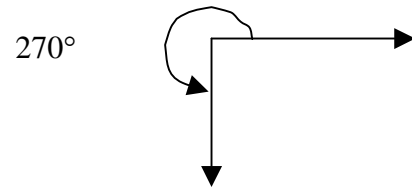
D. One and one-half of a right-angle turn.



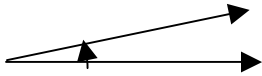
E. Two right-angle turns.



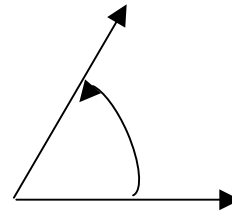
F. Three right-angle turns.



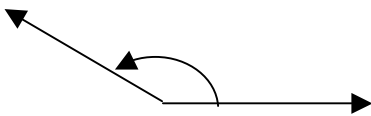
G. 20°



H. 70°



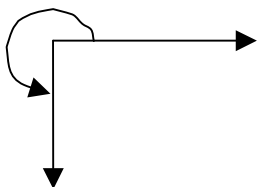
I. 150°



J. 180°



K. 270°



L. 360°

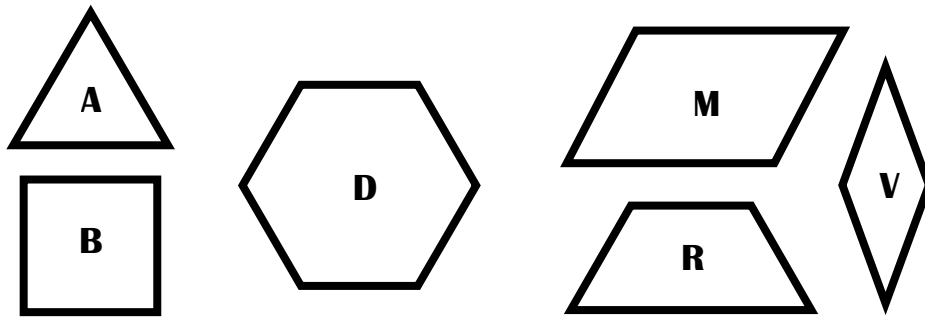


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Problem 3.3

In problem 3.2, you used a 90° angle as a benchmark, or reference, to help you sketch angles and estimate angle measures. The angles of some polygons in your Shapes Set can be other useful benchmarks.

Use your **Shape Set Labsheet** to estimate and record the **measure** of each angle of shapes A, B, D, M, R, and V.



Be sure you are measuring the angles on your **Shape Set Labsheet!** Also, be sure to measure the angles on pg. 28 in your *Shapes & Designs* book.

Letter	Name	Angle Measure
A	triangle	The measure of each angle is 60° .
B	square	The measure of each angle is 90° .
D	hexagon	The measure of each angle is 120° .
M	rhombus	The measure of each small angle is 30° . The measure of each large angle is 150° .
R	parallelogram	The measure of each small angle is 60° . The measure of each large angle is 120° .
V	trapezoid	The measure of each small angle is 60° . The measure of each large angle is 120° .

Follow-Up

Angle a = 135°

Angle b = 30°

Angle c = 180°

Angle d = 315°

Angle e = 90°

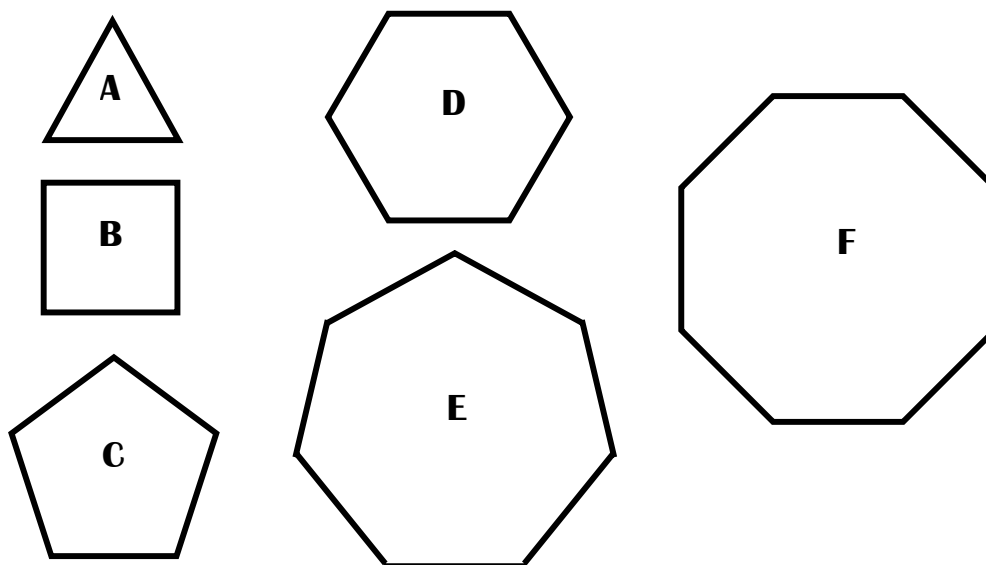
Angle f = 60°

Shapes & Designs Notes

Problem 4.1

The shape of a polygon depends on the number of sides it has, the length of those sides, and the size of its angles. In order to see how many sides and angles are related in polygons, you can gather some data from polygons, organize the data, and look for patterns. Patterns in the angle measures of regular polygons help to explain why hexagons show up in honeycombs.

Below are six regular polygons that are already familiar to you. All of the sides in all of the polygons are the same length. The angles where the sides meet are clearly not the same in all the figures. What pattern do you see in the sizes of the interior angles as the number of sides increases?



- A. Make a table that shows the name of each polygon, the number of sides it has, the measure of its angles and the sum of the measures of all its angles (this is called the angle sum).

Polygon	# of sides	Interior angle	Angle sum
<i>Triangle</i>	3	60°	180°
<i>Square</i>	4	90°	360°
<i>Pentagon</i>	5	108°	540°
<i>Hexagon</i>	6	120°	720°
<i>Heptagon</i>	7	128.5°	900°
<i>Octagon</i>	8	135°	1080°

- B. In your table, look for patterns that relate the number of sides a polygon has to the measure of its angles and to its angle sum. ***The measure of the interior angles increases with the number of sides. The angle sum increases by 180° with each additional side, starting from the triangle.***

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Problem 4.2

Definitions:

irregular polygons - polygons in which all the side lengths do not have the same measure

equilateral triangles - triangles where all sides are of equal measure

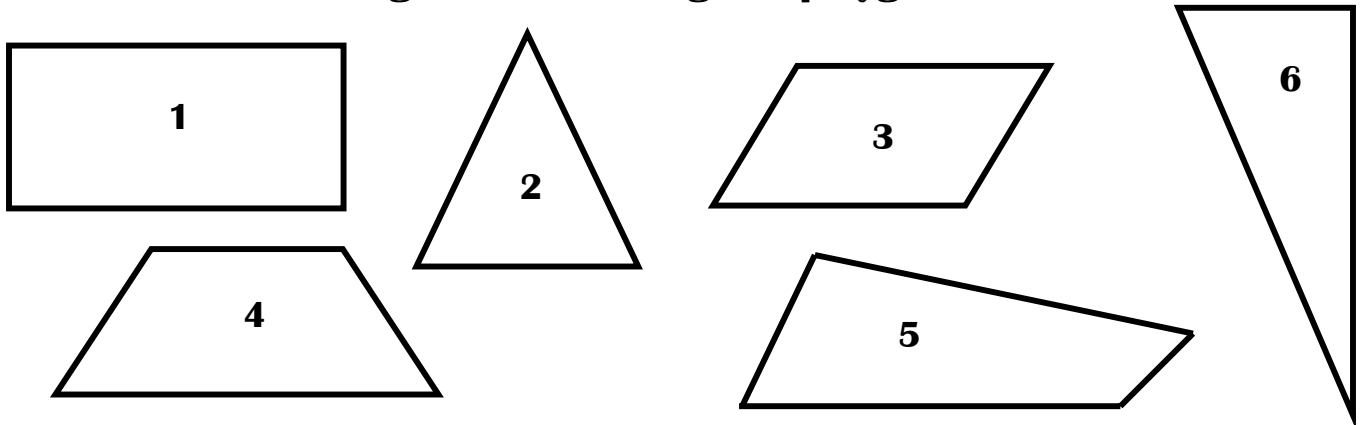
isosceles triangles - triangles with two sides of equal measure

In problem 4.1, you discovered that the angle sum of any regular polygon can be predicted easily from the number of sides it has.

Regular Polygon	# of sides	Angle sum	Regular Polygon	# of sides	Angle sum
<i>Triangle</i>	3	180°	<i>Hexagon</i>	6	720°
<i>Square</i>	4	360°	<i>Heptagon</i>	7	900°
<i>Pentagon</i>	5	540°	<i>Octagon</i>	8	1080°

In this problem, you will explore irregular polygons. These are polygons in which the sides are not all the same length.

Is there a relationship between the number of sides and the angle sum for irregular polygons?



- A. For each triangle and quadrilateral above, measure each interior angle and compute the angle sum. *The angle sum is 180° for each triangle and 360° for each quadrilateral.*
- B. How do the angle sums for these irregular polygons compare with the angle sums for the regular polygons? *The angle sums for irregular triangles and quadrilaterals are the same as for the regular triangles and quadrilaterals.*
- C. If you test the side-angle patterns you found by measuring the interior angles of some other triangles and quadrilaterals from your Shapes Set, you would get the same results.
- D. Use the information you have discovered about triangles and quadrilaterals to make a guess about the angle sums in irregular pentagons and hexagons. *The sum of the angle measures of an irregular pentagon is 540° . The sum of the angle measures of an irregular hexagon is 720° .*