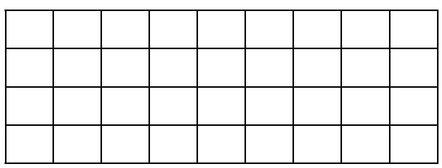
Covering & Surrounding Notes
Problem 1.1

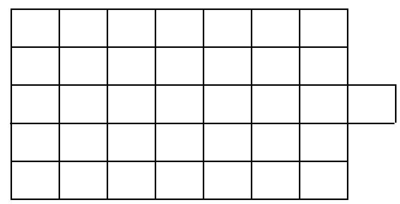
Definitions:
Area - the measure of the amount of surface enclosed by the sides of a figure.
Perimeter - the measure of the distance around a figure.

Bumper-cars is one of the most popular rides in traveling shows. A bumper car-ride includes the cars and a smooth floor with bumper rails around it. One company makes their bumper-car floors from tiles that are 1 meter by 1 meter squares. The bumper rail is built from sections that are one meter long. The company will often send sketches of their models to potential customers.

A. Badger State Shows in Wisconsin requested a bumper-car ride with a total of 36 square meters of floor space and 26 rail sections. ***Possible solutions:***

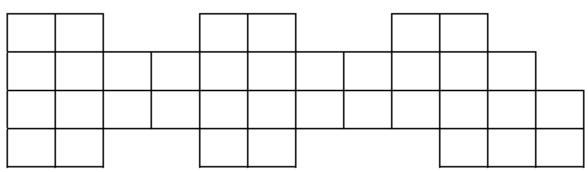


Area: ***36 square units***
Perimeter: ***26 units***



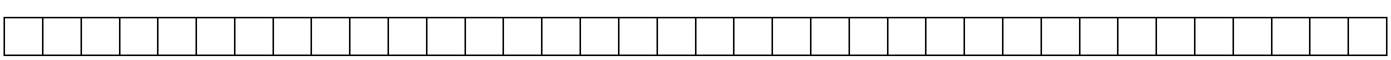
Area: ***36 square units***
Perimeter: ***26 units***

B. Lone Star Carnivals in Texas wants a bumper-car ride that covers 36 square meters of floor space and has lots of rail sections for riders to bump against. ***Possible solution:***



Area: ***36 square meters***
Perimeter: ***40 meters***

C. Design a bumper-car floor plan with 36 or more square meters of floor space that you think would make an interesting ride. ***Possible solution:***



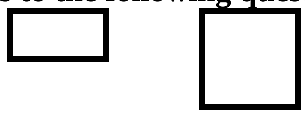
This figure creates a shape with an area of 36 square units and a perimeter of 74 units, but it does not make a very interesting bumper-car design.

Which measure - perimeter or area - better indicates the size of a bumper-car floor plan?

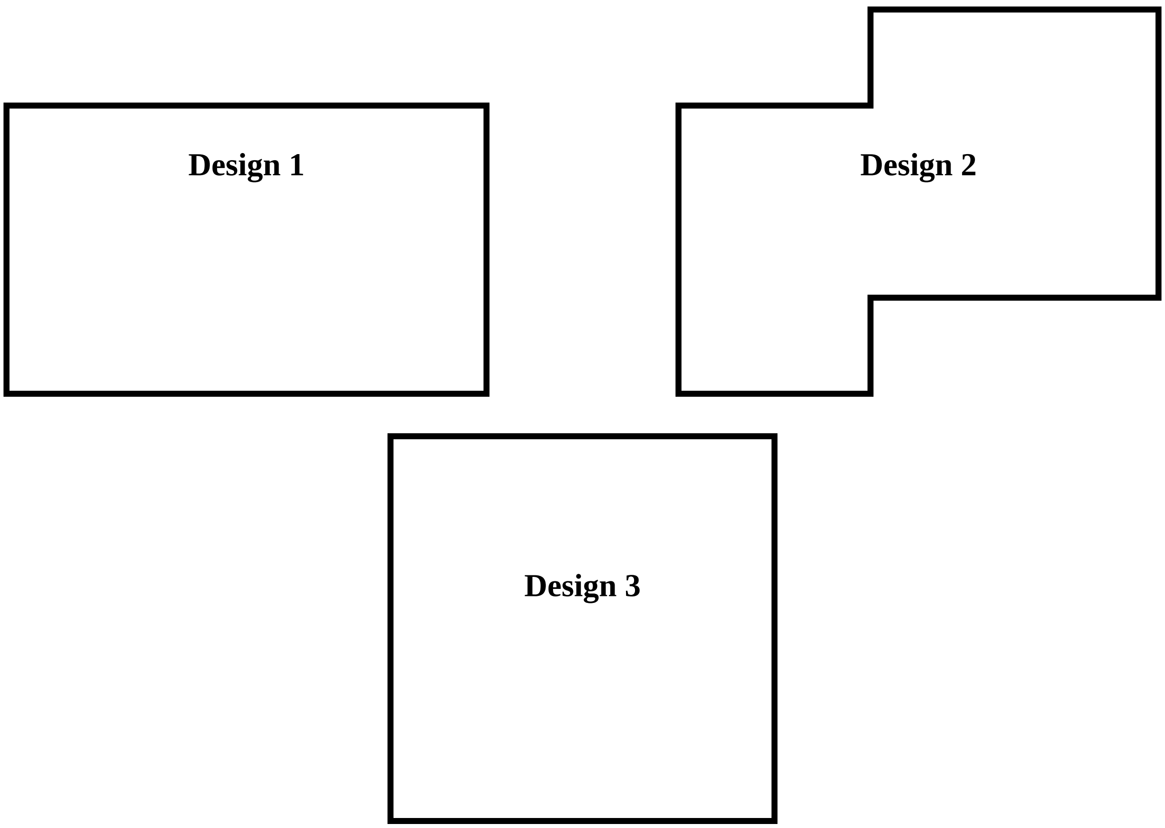
Area is probably a better measure, because it indicates the amount of space available for the bumper-cars.

Covering & Surrounding Notes
Problem 1.2

The MARs company advertises its carnival rides in a catalog. One section of the catalog shows bumper-car floor plans. The catalog shows only outlines of the plans, not the grid of the floor tiles or the rail sections. Below are three of the designs shown in the catalog. Using the sample floor tile section and the sample rail section, determine the answers to the following questions.



rail section **floor tile section**



A. Which of the three designs has the greatest floor space (has the greatest area)?

Design 1: area is 15 units square.
Design 2: area is 15 units square.
Design 3: area is 16 units square. ✓

B. Which of the three designs requires the most rail sections (has the greatest perimeter)?

Design 1: perimeter is 16 units.
Design 2: perimeter is 18 units. ✓
Design 3: perimeter is 16 units.

Covering & Surrounding Notes
Problem 1.3

The designers at MARs specialize in creating unusual floor plans for bumper-car rides. But when it comes time to prepare estimates or bills for customers, they turn the plans over to the billing department.

Refer to the designs on the back of this sheet or on pg. 10 and 11 in your book.

A. The MARs company charges $25 for each rail section and $30 for each floor tile. How much would each of the designs cost?

Examples:

Calculations for Design A Cost of Tiles:

9 tiles x $30 for each tile

9 x $30 = $270

Calculations for Design A Cost of Rails:

12 rails x $25 for each rail

12 x $25 - $300

Design	Area	Perimeter	Cost of tiles	Cost of rails	Total cost
A	9 square meters	12 meters	$270	$300	$570
B	7 square meters	12 meters	$210	$300	$510
C	9 square meters	20 meters	$270	$500	$770
D	12 square meters	14 meters	$360	$350	$710
E	9 square meters	14 meters	$270	$350	$620
F	9 square meters	16 meters	$270	$400	$670
G	11 square meters	24 meters	$330	$600	$930
H	9 square meters	20 meters	$270	$500	$770
I	10 square meters	16 meters	$300	$400	$700
J	16 square meters	34 meters	$480	$850	$1330

B. If you were the buyer for an amusement company, which design would you choose?

When making your decision, you should consider both cost and design.

Follow-Up

1. Of all the designs, which have an area of 9 square meters? *A, C, E, F, and H*

2. Give the price (total cost) of each design you listed in question 1.

   design A: $570, design C: $770, design E: $620, design F: $670, design H: $770

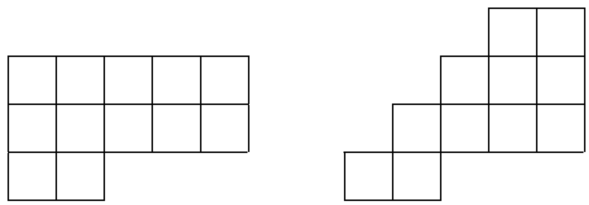
3. What accounts for the difference in the prices of the designs you listed in question 1?

   The designs have different perimeters. Larger perimeters require more rail sections and, as a result, cost more to build.

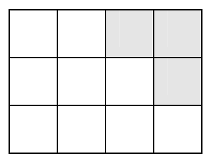
Covering & Surrounding Notes
Problem 1.4

Five of the bumper-car designs in Problem 1.3 had an area of 9 square meters. You found that these designs had different prices because their perimeters were different. Questions A-E refer to the designs from Problem 1.3

A. Build a design with the same area as design G, but with a smaller perimeter***. Several figures can be constructed with an area of 11 square meters and a perimeter of less than 24 meters. Possible answers:***



B. Design E can be made from design D by removing three tiles. ***You can remove the shaded squares to get design E from design D.***



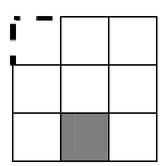
How does the area of design D compare to the are of design E? ***The area of design D (12 square meters) is three more than the area of design E (9 square meters).***

How does the perimeter of design D compare to the perimeter of design E? ***The perimeter of both designs is 14 meters.***

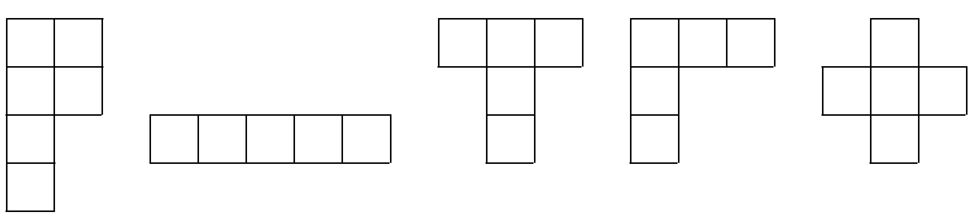
C. Design F and design I have the same perimeter. Can you rearrange the tiles of design F to make design I? ***No, you cannot rearrange design F to get design I because they are made from different numbers of tiles.***

D. Design A and design C have the same area. Can you rearrange the tiles of design A to make design C? ***Yes, the figures are made from the same number of tiles.***

E. Look a design B. Can you move one tile to make a new design with a perimeter of 14 units? ***Possible solution: Move the shaded square as shown, producing a new figure of area 7 square meters, but perimeter of 14 meters.***

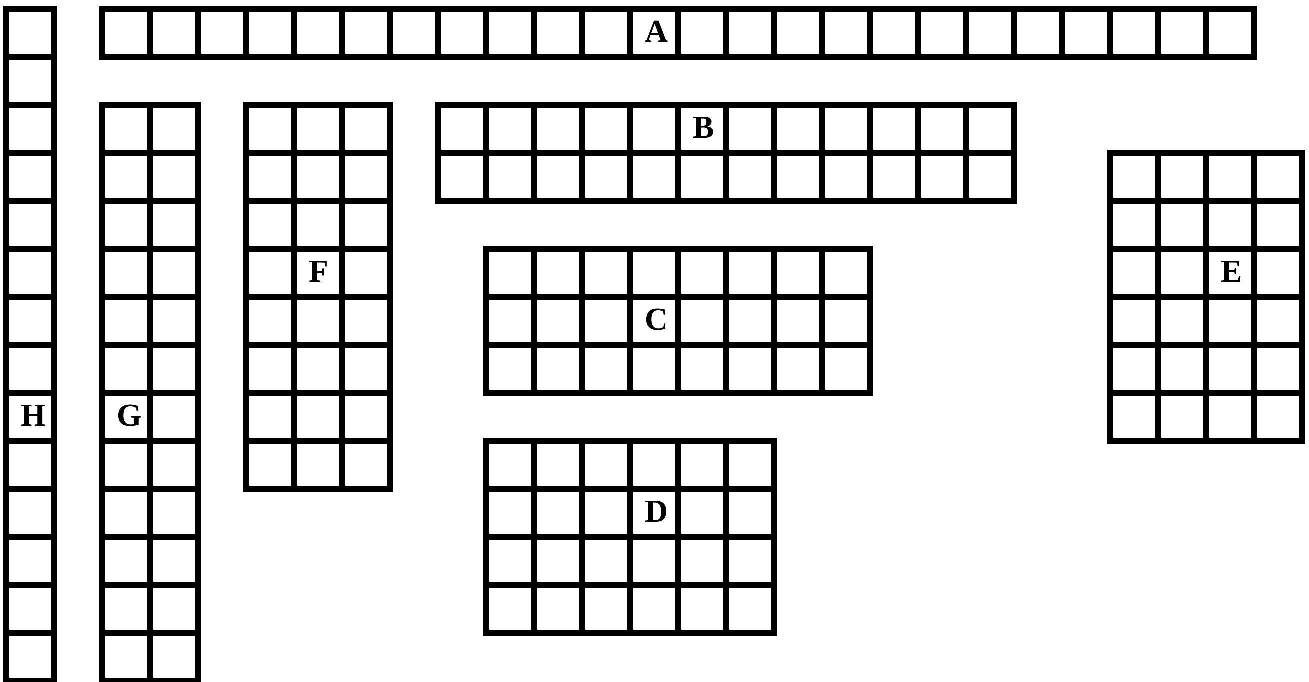


If two tile designs have the same area and the same perimeter, must they look exactly alike? ***Having the same area and the same perimeter does not guarantee that two designs will be the same. For example, all of these figures have an area of 5 and a perimeter of 12:***



Covering & Surrounding Notes
Problem 3.1

The rangers in the Great Smoky Mountains National Park want to build several inexpensive storm shelters. The shelters must have 24 square meters of floor space. Suppose that the walls are made of sections that are 1 meter wide and cost $125.



Sketch	Length	Width	Perimeter	Area	Cost
A	*1 meter*	*24 meters*	*50 meters*	*24 sq*	*$6250*
B	*2 meters*	*12 meters*	*28 meters*	*24 sq*	*$3500*
C	*3 meters*	*8 meters*	*22 meters*	*24 sq*	*$2750*
D	*4 meters*	*6 meters*	*20 meters*	*24 sq*	*$2500*
E	*6 meters*	*4 meters*	*20 meters*	*24 sq*	*$2500*
F	*8 meters*	*3 meters*	*22 meters*	*24 sq*	*$2750*
G	*12 meters*	*2 meters*	*28 meters*	*24 sq*	*$3500*
H	*24 meters*	*1 meter*	*50 meters*	*24 sq*	*$6250*

B. Based on the cost of the wall sections, which design would be the least expensive to build?

The 4 x 6 (or 6 x 4) shelter is the least expensive to build. This floor plan is the most square-like of the possibilities. The shelter would have the most open space and the fewest wall sections.

C. Which shelter plan has the most expensive set of wall sections?

The 1 x 24 (or 24 x 1) shelter is the most expensive to build. The floor plan is long and skinny, with the least open space and the most wall sections.

Covering & Surrounding Notes
Problem 3.2

I problem 3.1 you worked with rectangles to help you understand the relationship between area and perimeter. In this problem you will look at what happens when you cut an interesting part from a rectangle and slide that piece onto another edge.

Draw a 4 x 6 rectangle.

Starting at one corner, cut out an interesting path to an adjacent corner.

Slide the piece you cut onto the opposite edge.

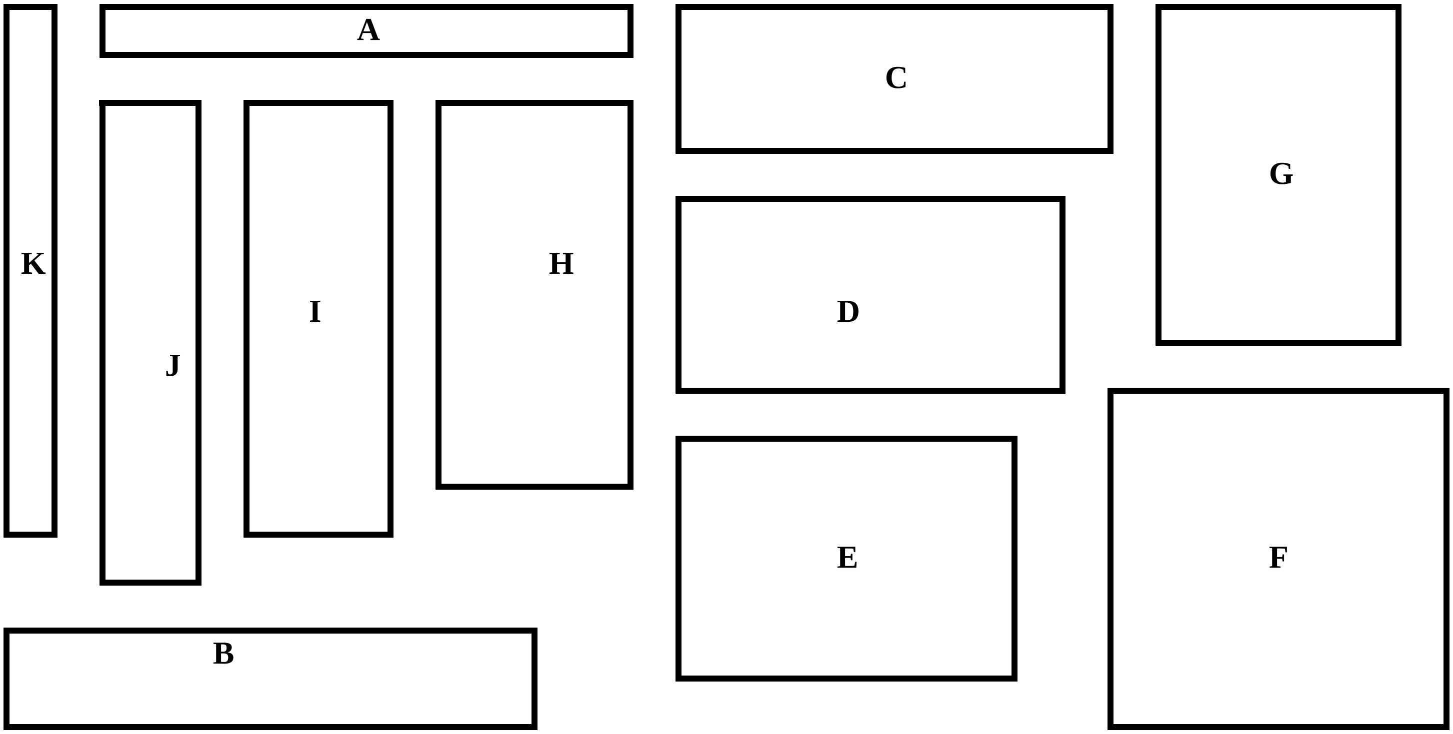
A. Find the area and the perimeter of your new figure. ***Areas will all be 24 square units; perimeters will be 20 units or greater.***

B. Is the perimeter of the new figure larger than, the same as, or smaller than the perimeter of a 4 x 6 rectangle? ***The perimeter of the rectangle is 20 units. The perimeters of the new figures are all greater than 20 units, because when you cut and move the piece, more edge is exposed.***

C. Could you make a figure with an area of 24 square units with a longer perimeter than you found in your first figure? ***Yes, by removing a piece from the rectangle with a longer cut edge.***

Covering & Surrounding Notes
Problem 4.1

Suppose you want to help a friend build a rectangular pen for her dog, Shane. You have 24 meters of fencing, in 1-meter lengths, to build the pen. Which rectangular shape would be best for Shane? Below are all the possible rectangles with a perimeter of 24 meters.



Rectangle	Length	Width	Perimeter	Area
A	*1 meter*	*11 meters*	*24 meters*	*11 square meters*
B	*2 meters*	*10 meters*	*24 meters*	*20 square meters*
C	*3 meters*	*9 meters*	*24 meters*	*27 square meters*
D	*4 meters*	*8 meters*	*24 meters*	*32 square meters*
E	*5 meters*	*7 meters*	*24 meters*	*35 square meters*
F	*6 meters*	*6 meters*	*24 meters*	*36 square meters*
G	*7 meters*	*5 meters*	*24 meters*	*35 square meters*
H	*8 meters*	*4 meters*	*24 meters*	*32 square meters*
I	*9 meters*	*3 meters*	*24 meters*	*27 square meters*
J	*10 meters*	*2 meters*	*24 meters*	*20 square meters*
K	*11 meters*	*1 meter*	*24 meters*	*11 square meters*

Which design would give Shane the best pen for running?

1 x 11 would give the longest running space, but the 2 x 10 pen would give Shane more room to turn around.

Which design would give Shane the most space for playing?

The 6 x 6 pen has the largest area and will give Shane the most playing space.

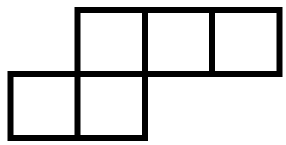
Covering & Surrounding Notes
Problem 4.2

Definition:

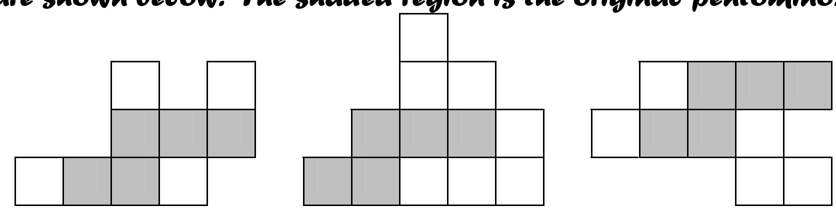
pentomino - shape made from five identical square tiles connected together along their edges.

In Problem 4.1, you explored the relationship between area and perimeter by investigating rectangles that could be made with a fixed perimeter of 24 unit. In this problem you will continue to investigate fixed perimeter by adding tiles to a pentomino.

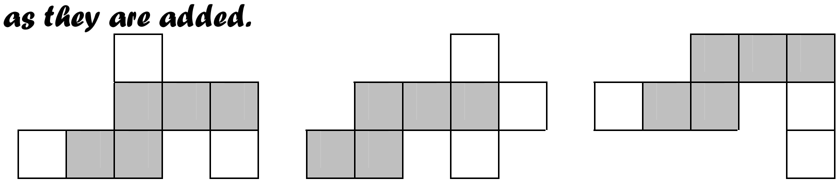
Begin by making this pentomino.



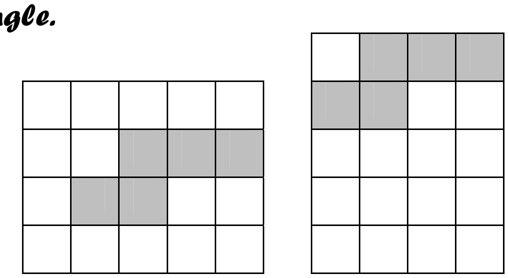
A. Add tiles to the pentomino to make a new figure with a perimeter of 18 units. ***Some possible arrangements are shown below. The shaded region is the original pentomino.***



B. What is the smallest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? ***The smallest number of pentominos would be three. Some possible arrangements are shown below. The shaded region is the original pentomino. Each of the three tiles much touch only one edge as they are added.***



C. What is the largest number of tiles you can add to the pentomino to make a new figure with a perimeter of 18 units? ***The largest number of pentominos would be 15. Some possible arrangements are shown below. The shaded region is the original pentomino. Each figure must enclose the pentomino in a 4 x 5 rectangle.***



How does adding one tile change the perimeter of a figure? ***Depending on where you add the new tile, the perimeter can increase by 2 units, stay the same, or decrease by 2 units.***

Covering & Surrounding Notes
Problem 5.1

Definition:
Area of a rectangle - can be found by multiplying the length by the width

You have found areas and perimeters of both rectangular and nonrectangular shapes. There are two methods you may use for finding area. They are:

- ★ **count the numbers of squares enclosed by a rectangle**
- ★ **multiply the number of rows by the numbers of columns**

Estimate the area for each shape on Labsheet 5.1.

Possible Solutions for Problem 5.1

Figure	Area	Explanation
A	13 ½ sq. units	There are 12 whole squares and 3 half squares.
B	8 sq. units	Cut off a triangle. Move it to the other side to make a 2 x 4 rectangle.
C	19 ¼ sq. units	There are 15 whole squares, 8 half squares, and 1 fourth square.
D	18 sq. units	Take a 3 x 8 rectangle and subtract a 2 x 3 rectangle.
E	35 sq. units	There are 28 whole squares and a triangle on each side. Each triangle is half of a 1 x 7 rectangle, or 3 ½ square units. Together the triangle is 7 square units.
F	9 sq. units	There are 8 whole squares and 2 half squares.
G	8 sq. units	Make a vertical cut in the middle and slide the pieces together to make a 4 x 2 rectangle.

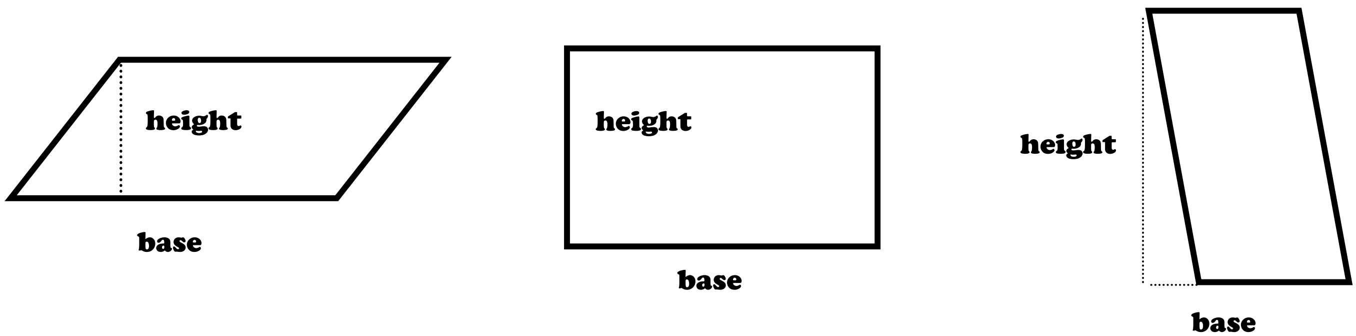
Covering & Surrounding Notes
Problem 5.2

Definition:

base - the side of a polygon on which the figure rests

height - the distance from the base of the polygon to the top of the polygon

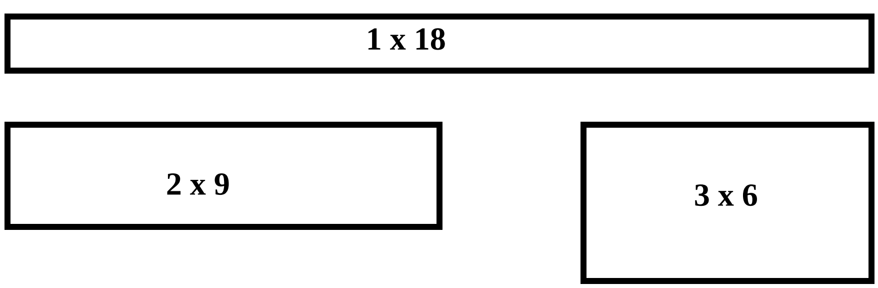
Parallelograms are often described by giving their *base* and *height*. These drawings illustrate the meanings of these terms.



You can think of the height as the distance a rock would fall if you dropped it from a point at the top of a parallelogram down to the line that is the base. If the first parallelogram, if we dropped the rock from the upper-left corner, it would fall inside the parallelogram. In the second parallelogram (a rectangle), the rock would fall along one of the sides. In the third parallelogram, if we dropped the rock from the upper-left corner, it would fall outside the parallelogram.

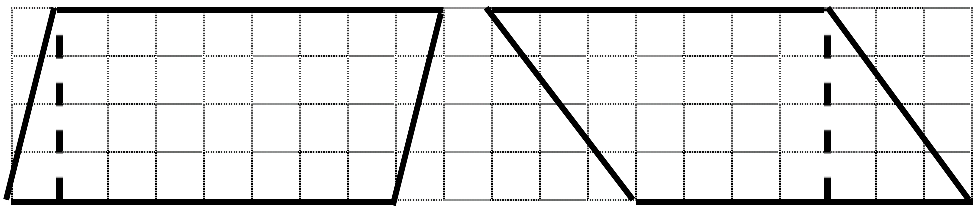
In this problem, you will draw parallelograms that meet given requirements or constraints.

A. Draw a rectangle with an area of 18 cm^2. Try to draw a different rectangle with an area of 18cm^2. Do the rectangles have the same perimeter? ***Each rectangle has a different perimeter.***

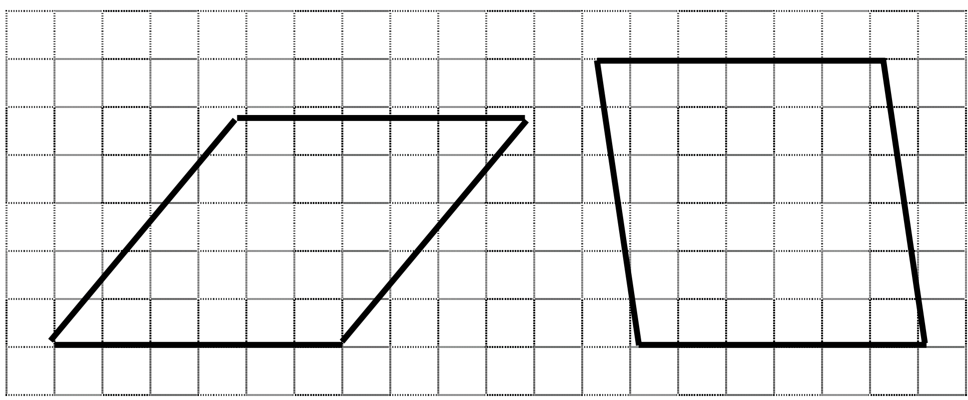


B. Draw a rectangle with the dimensions 3 cm by 8 cm. Try to draw a different rectangle with these same dimensions. Do the rectangles have the same area? ***It is not possible to draw two different rectangles. You can draw the same rectangle, but with different orientations (going in different directions).***

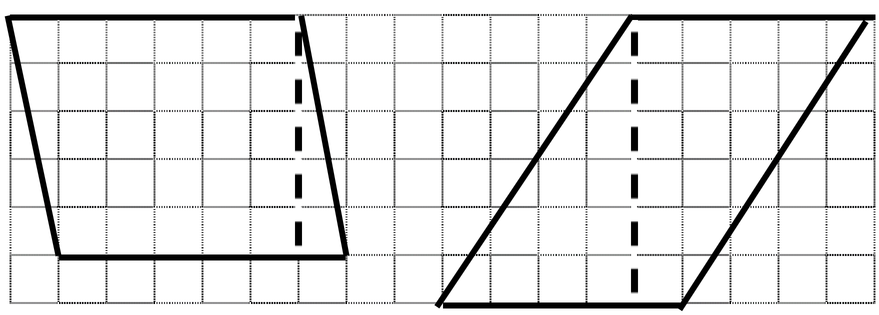
C. Draw a parallelogram with a base of 7 cm and a height of 4 cm. Try to draw a different parallelogram with these same dimensions. Do these parallelograms have the same area? ***All of the parallelograms will have an area of 28 square units.***



D. Draw a parallelogram with all sides lengths equal to 6 cm. Try to draw a different parallelogram with all side lengths equal to 6 cm. Do the parallelograms have the same area? ***The areas of the parallelograms will vary from 0 to 36 square cm.***



E. Draw a parallelogram with an area of 30 cm^2. Try to draw a different parallelogram with the same area. Do the parallelograms have the same perimeter? ***The base times the height of all the parallelograms will be 30 square cm. The perimeters will vary.***



Formula for finding area of parallelogram is:

Area = base x height

Covering & Surrounding Notes
Problem 6.1

You can always find the area of a figure by overlaying a grid and counting squares, but you probably realize that this can be very time-consuming.

Estimate the area and perimeter for each shape on Labsheet 6.1

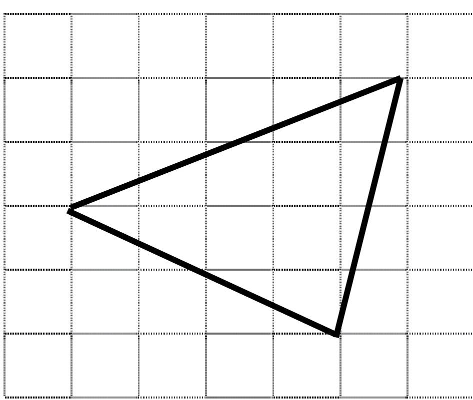
Possible Solutions for Problem 6.1

Figure	Area	Perimeter
A	12 sq. units	about 17 units
B	35 sq. units	about 29 units
C	12 sq. units	about 19 ½ units
D	27 sq. units	about 24 units
E	16 sq. units	about 19 ½ units
F	30 sq. units	about 26 ½ units
G	25 ½ sq. units	about 24 ½ units
H	21 sq. units	about 21 units

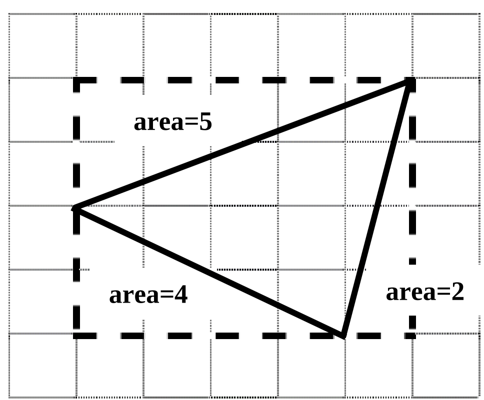
Follow-Up

Find the area and perimeter of this triangle.

Original figure



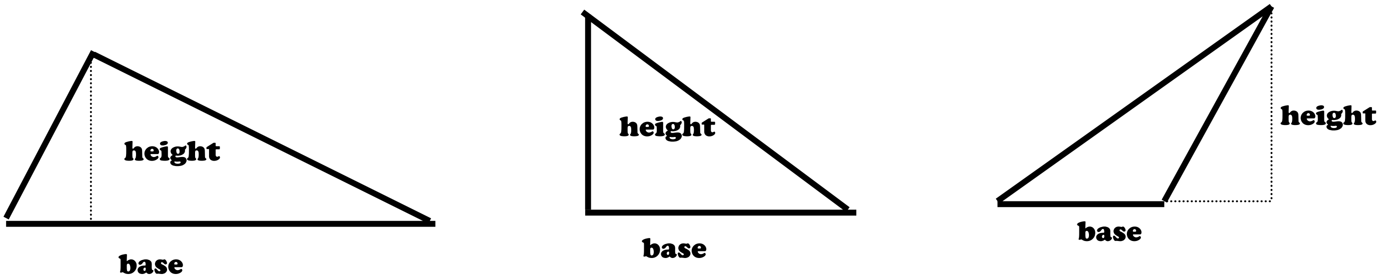
Possible solution



The area of the large rectangle is 20 sq. units.
If you subtract the area of each section outside the triangle, but in the rectangle, you would have:
20 - 5 - 4 - 2 = 9 square units

Covering & Surrounding Notes
Problem 6.2

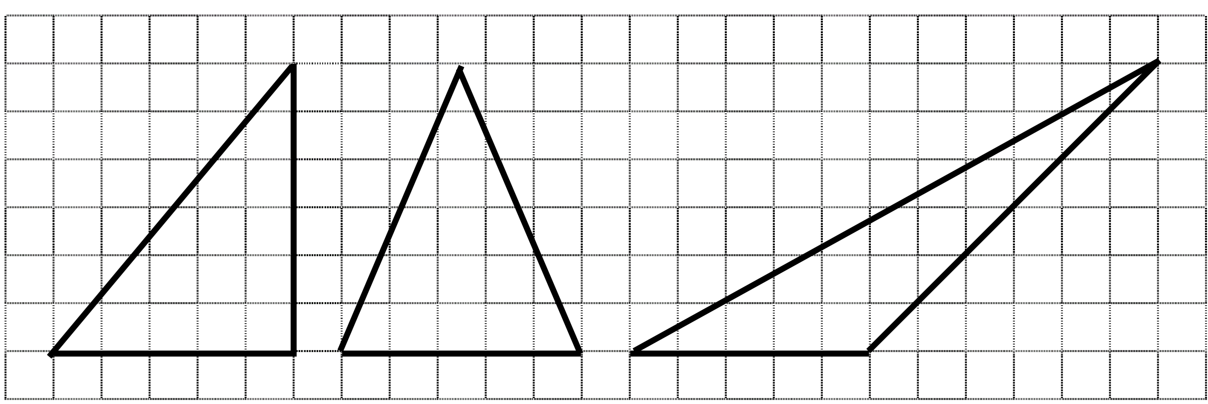
As with parallelograms, triangles are often described by giving their *base* and *height.* These drawings illustrate the meanings of these terms.



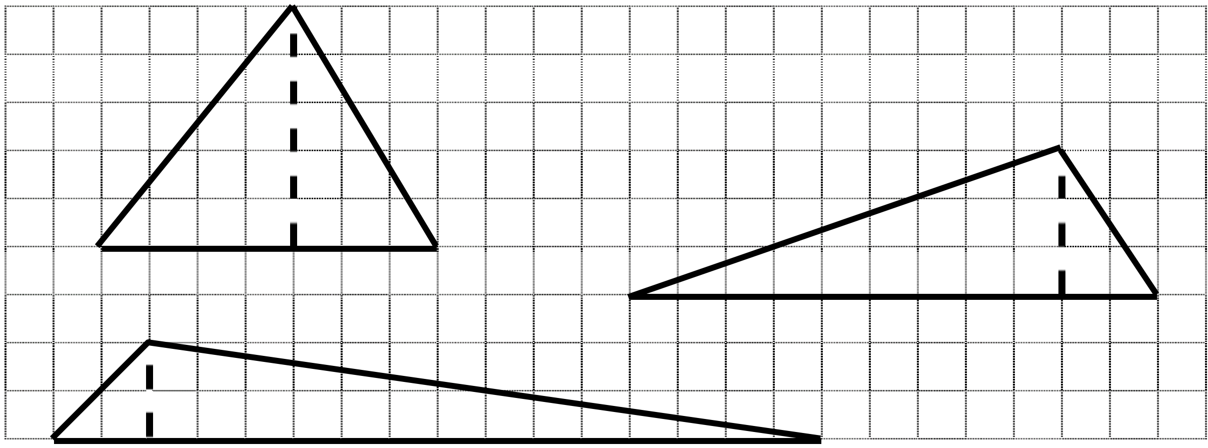
You can think of the height as the distance a rock would fall if you dropped it from a point at the top vertex of the triangle down to the line that the base is on. In the first triangle, the height falls inside the triangle. In the second triangle, the height is one of the sides. In the third triangle, the height falls outside the triangle.

In this problem, you will draw triangles that meet given constraints.

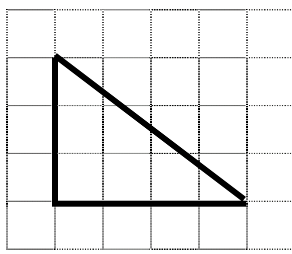
A. Draw a triangle with a base of 5 cm and a height of 6 cm. Try to draw a different triangle with the same dimensions. Do the triangles have the same area? ***There are an infinite number of possibilities. All of the areas are the same, but the perimeters are different.***



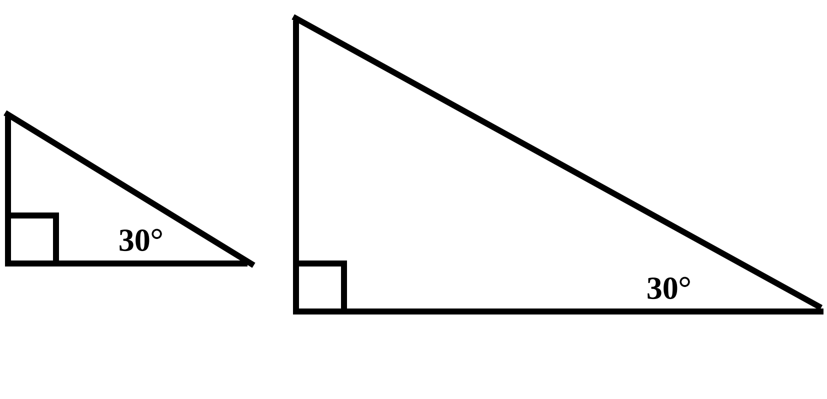
B. Draw a triangle with an area of 15 cm^2. Try to draw a different triangle with the same area. Do the triangles have the same perimeter? ***The perimeters will be different.***



C. Draw a triangle with sides of length 3 cm, 4 cm, and 5 cm. Try to draw a different parallelogram with these same side lengths. Do these triangles have the same area? ***Only one triangle is possible with the given side lengths.***



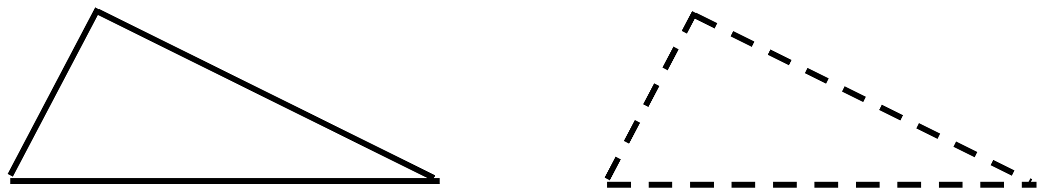
D. A right triangle is a triangle that has a right angle. Draw a right triangle with a 30° angle. Try to draw a different right triangle with a 30° angle. Do the triangles have the same area? ***You can draw an infinite number of triangles with a 30°angle. The triangles will have different areas and different perimeters, but they will all have the same angle measures and the same shape.***



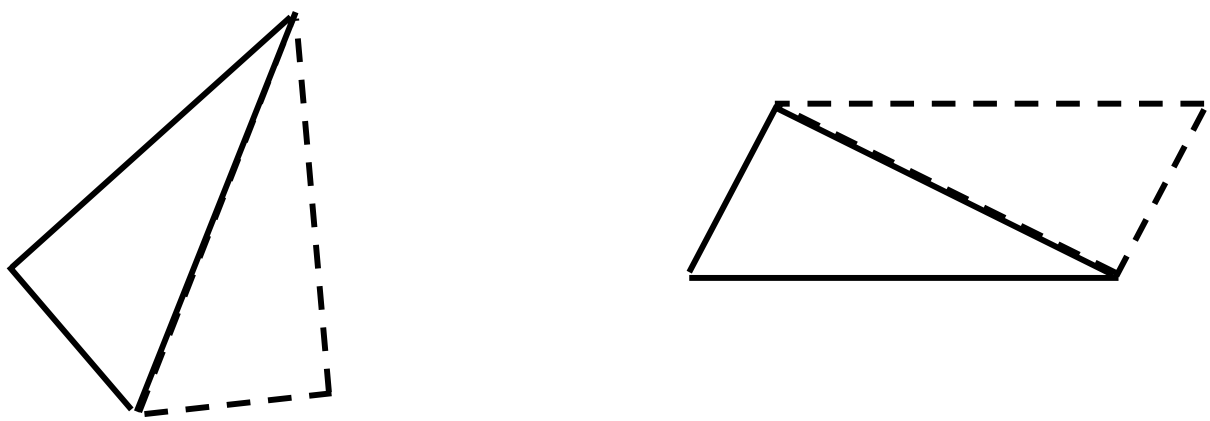
Covering & Surrounding Notes
Problem 6.3

Remember that the *height* of a triangle is the perpendicular distance from one vertex to the opposite side. The height can fall inside or outside the triangle and may be the length of one of the sides.

Look at these triangles. Notice that they are they same shape and size.



If you take both triangles and put them together, you can create a polygon.



Some observations:

- ★ **You can always make a parallelogram from two identical triangles.**
- ★ **The edge lengths of the parallelogram are two of the three side lengths of the triangle.**
- ★ **Two identical triangles make a parallelogram.**
- ★ **The area of a triangle is half the area of a parallelogram.**

You can find the area of a triangle by finding the area of a parallelogram and taking half.

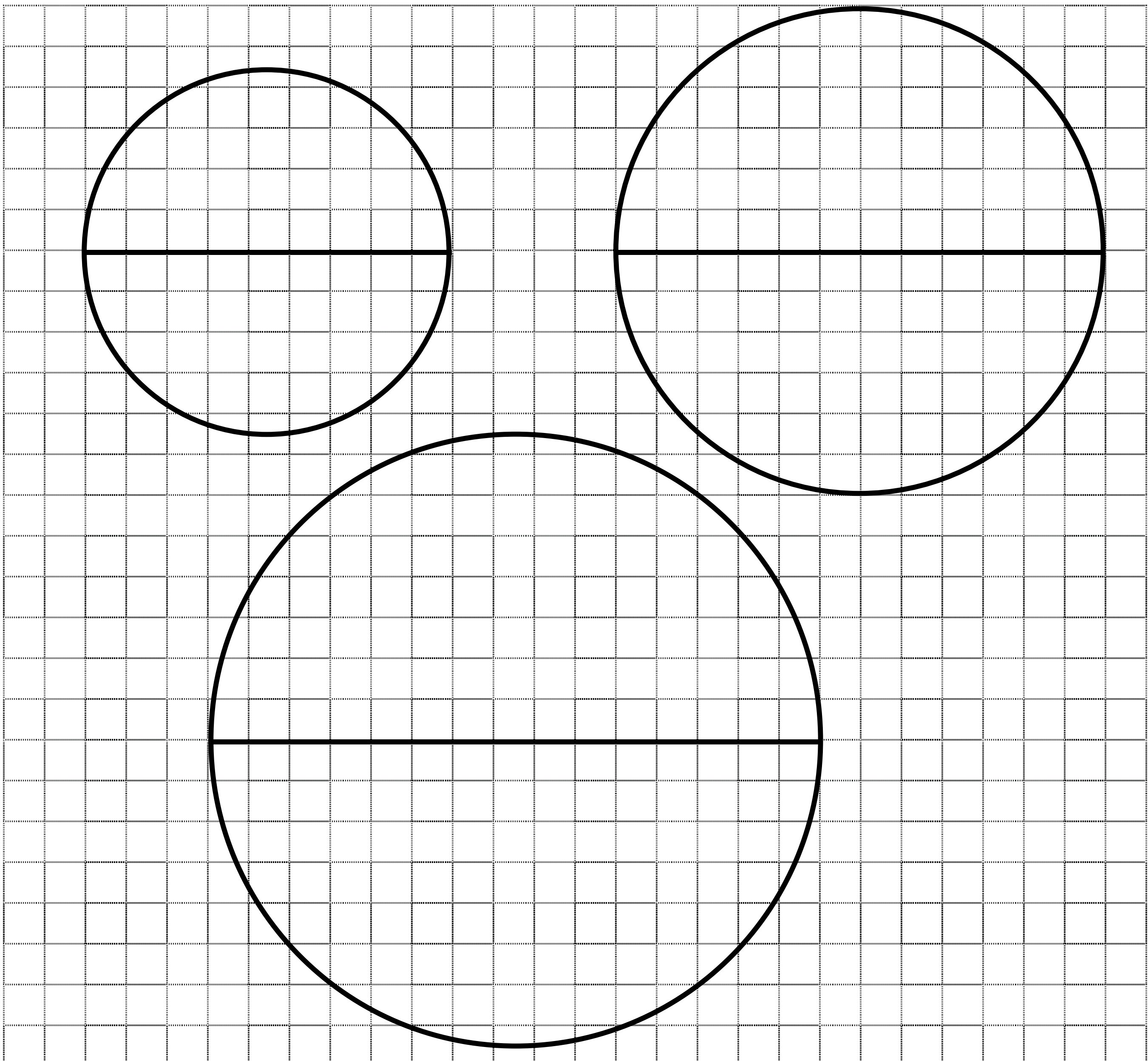
Formula for finding the area of a parallelogram is: *area = base x height*

Formula for finding the area of a triangle is: *area = (base x height) ÷ 2*

Covering & Surrounding Notes
Problem 7.1

The Sole D'Italia Pizzeria sells small, medium, and large pizzas. The cheese pizza is the most popular. A small pizza has a diameter of 9 inches and cost $6, a medium has a diameter of 12 inches and costs $9, and a large has a diameter of 15 inches and costs $12.

A. Each size pizza is shown below. Let one square of the grid represent 1 square inch on the pizza. Estimate the radius, circumference, and area of each pizza. (You might want to use string to measure the circumference)



As a review:

Diameter - any line segment that extends from a point on the circle, through the center, to another point on the circle

Radius - any line segment from the center to a point on the circle (half the length of the diameter)

Circumference - the distance around the circle (like the perimeter of a polygon)

Area - the space occupied within the circle

Review the following table.

Size	**Diameter** *(across the circle)*	**Radius** *(half the diameter)*	**Circumference** *(distance around)*	**Area** *(space inside)*
S	9 inches	4.5 inches	about 28 inches	about 64 sq. inches
M	12 inches	6 inches	about 38 inches	about 113 sq. inches
L	15 inches	7.5 inches	about 47 inches	about 177 sq. inches

B. Which measurement - radius, diameter, circumference, or area - seems most closely related to price?

The diameter is most closely related to the price, because as the diameter changes by 3 inches, the price changes by $3.

Be sure to go back to the figures on the front of the page and label the measures for the diameter, radius, circumference, and area for each size pizza.

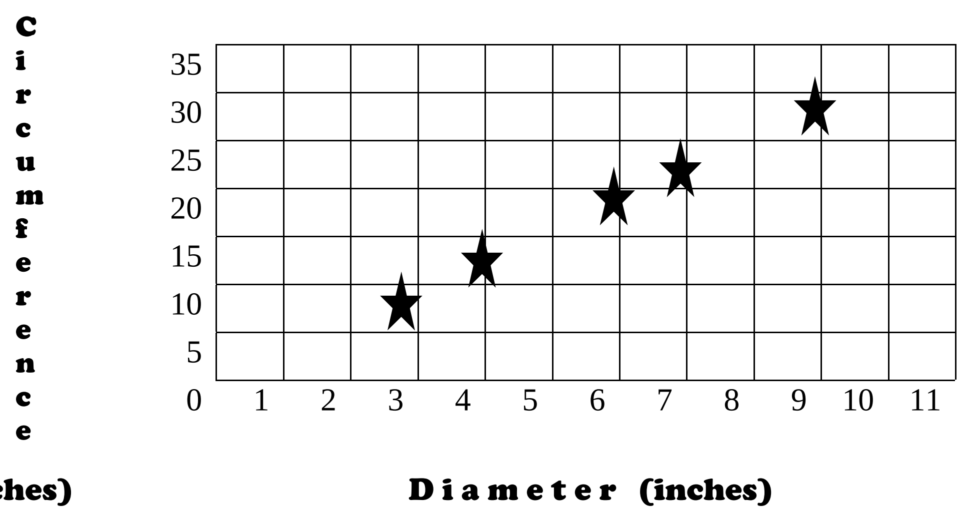
Covering & Surrounding Notes
Problem 7.2

Mathematicians have found a relationship between the diameter and the circumference of a circle. You can try to discover this relationship by measuring many different circles and looking for patterns. The patterns you discover can help you develop a short cut for finding the circumference of a circle.

A. Below are the results of several objects that were measured.

Object	Distance Across (*diameter*)	Distance Around (*circumference*)	*Circumference ÷ Diameter*
soda can	*2.75 inches*	*8.5 inches*	*8.5 ÷ 2.75 ≈ 3.09*
Clock	*6.75 inches*	*21.25 inches*	*21.25 ÷ 6.75 ≈ 3.15*
paper plate	*9 inches*	*28 inches*	*28 ÷ 9 ≈ 3.11*
coffee can	*4 inches*	*12.5 inches*	*12.5 ÷ 4 ≈ 3.13*
potted plant	*6 inches*	*19 inches*	*19 ÷ 6 ≈ 3.17*
Table	*42 inches*	*132 inches*	*132 ÷ 42 ≈ 3.14*

B. Make a coordinate graph of your data. Use the horizontal axis for diameter and the vertical axis for circumference.



C. What do you think the relationships is between the diameter and the circumference of a circle?

The circumference is a little more than 3 times the diameter. This is also called "pi" and is written using the pi symbol - π

1. How can you find the circumference of a circle if you know its diameter?

Add together 3 diameters and a bit more, or multiply the diameter by 3 and add a bit more.

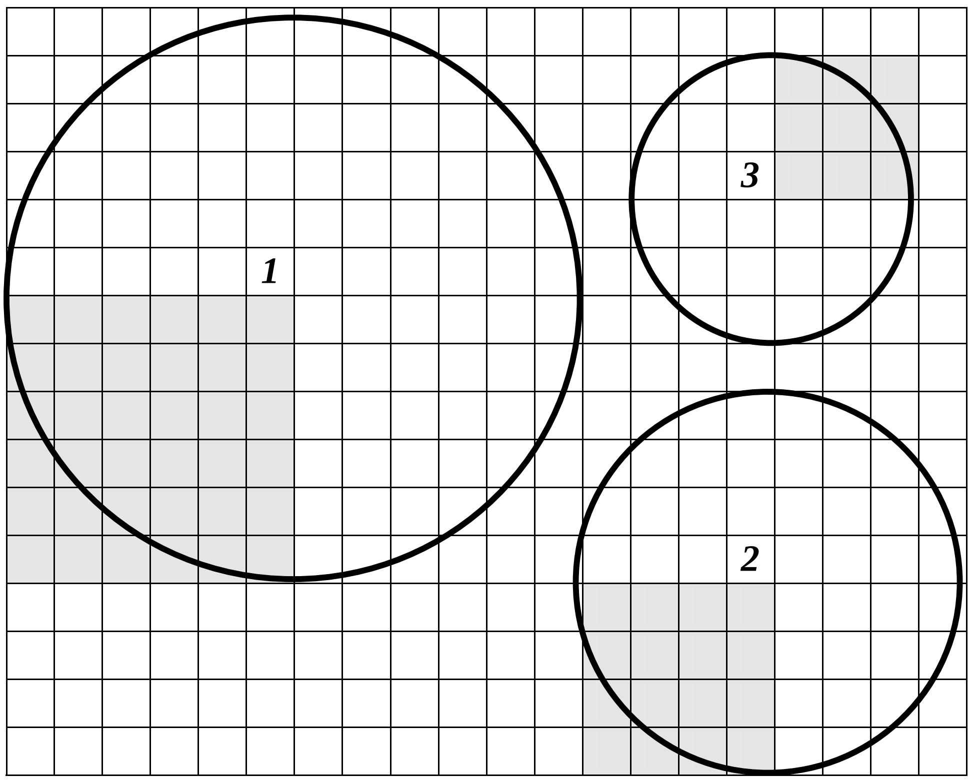
2. How can you find the diameter of the circle if you know its circumference?

Divide the circumference by 3 and subtract a bit from the result.

Covering & Surrounding Notes
Problem 7.4

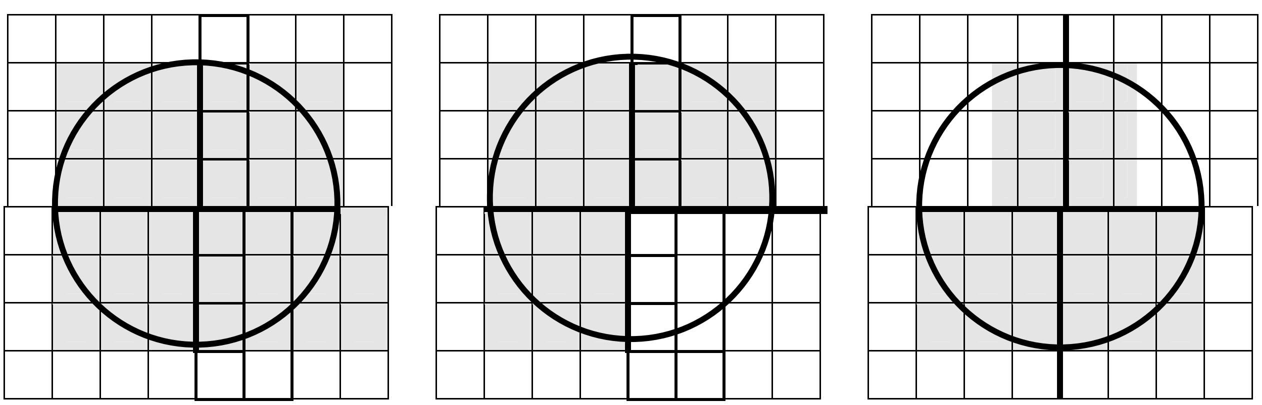
In Investigations 5 and 6, you learned some things about parallelograms and triangles by comparing them to rectangles. Now you will find out more about circles by comparing them to squares.

A portion of each circle is covered by a shaded square. The sides of each shaded square are the same length as the radius of the circle. We call such a square a "radius square."



A. For each circle, try cutting out radius squares from a sheet of grid paper and trying to cover the circles on Labsheet 7.4. You may cut the radius square into parts if you need to.

Possible solutions:



four squares is too many *three squares will be enough plus a little extra*

Circle	Radius of circle	Area of radius square	Area of Circle	Number of radius squares needed
1	*6 units*	*36 square units*	*about 113 square units*	*a bit more than 3*
2	*4 units*	*16 square units*	*about 50 square units*	*a bit more than 3*
3	*3 units*	*9 square units*	*about 28 square units*	*a bit more than 3*

C. Describe any patterns you see in your data.

It takes a little more than three radius squares to cover any circle.

D. If you were asked to estimate the area of any circle in "radius squares," what would you report as the best estimate?

The area of any circle is a little bit more than three radius squares.

Summary:

- **the area of circle is a little bit more than 3 times the area of a square that has the circle's radius as its side length.**
- **The area of the radius square is found by multiplying the length by the width which is the radius times the radius.**
- **The area of a circle is (a little bit more than 3) times the radius times the radius or (a little bit more than 3) times radius square**
- **A little bit more than 3 is also known as "pi" and is written using the symbol π. It has an approximate value of 3.14.**

1. How can you find the area of a circle if you know the diameter or the radius?

To find the area, find the radius and the area of a radius square. The area of the circle is the area of the radius square times 3.14.

2. How can you find the diameter or radius of a circle if you know the area?

To find the radius, divide the area by 3.14 to get the area of a radius square. Then find a number whose square is the area of the radius square. Double the radius to give the diameter.

Formula for area of a circle in words is:

Area of a circle = radius x radius x pi

OR

Area of a circle = radius² x pi

Formula for area of a circle in symbols is:

$A = r \times r \times \pi$ OR $A = r^2 \times \pi$